Mathematics Enhancement Programme (MEP)

PRIMARY Book 4

Support for Parents, Carers and Teachers

Yearly Objectives with Examples

Key Facts and Glossary

Problem Solving Examples

Overview of Contents

Detailed Contents

Mathematics Enhancement Programme

TEACHING SUPPORT

GENERAL OVERVIEW

This initiative, the *Mathematics Enhancement Programme, Primary Project*, has been developed by the Centre for Innovation in Mathematics Teaching (CIMT) at Plymouth University to enhance the mathematical progress of learners in primary schools. It is based on evidence of good practice from mathematically high performing countries.

MEP aims to support these KEY STRATEGIES for effective mathematics teaching and learning:

1. **Lesson** well prepared (teacher knows the lesson plan well and is aware of problems/difficulties that are likely to occur), resources are at hand, board prepared in advance, learners have their own resources on desk.

2. **Seating**: learners have eye contact with the teacher and can get to the board quickly and easily.

3. **Whole-class interactive teaching** predominates, with planned intervals of individual and paired work. Learners on task and given the chance to demonstrate, answer, explain to the class.

4. **Friendly, non-confrontational atmosphere** where learners learn from and support others and have fun! Mistakes used as teaching points. Encouragement given to those who have difficulty and praise given when deserved. Learners are encouraged to appreciate the good work of others.

5. **Spiral curriculum** with continual revision; learning by heart encouraged, with progression in small, logical steps.

6. **Visualisation** and **manipulatives** are used in the early years and less able learners. Relating contexts to learners' experiences, demonstrate on number lines, modelling to help understanding.

7. **Exercises** reviewed interactively with the whole class at the time. Learners give the solutions, not the teacher, and rest of the class agrees/disagrees or suggests alternative solutions. Learners are expected to correct their own work. Teacher gives hints only if the whole class is stuck.

8. **Challenges** or **extension work** set for able learners, or they help less able neighbours.

9. Introductory and Reinforcement tasks to help ensure that no children are left behind.

10. **Correct notation, layout** and **language** used at all times. Teacher acts as a model for learners to follow (on board and orally), repeating/showing a learner's explanation more clearly and succinctly where necessary.

11. **Good pace** and **varied activities** related to the concept being taught. Time limits set for individual/paired work. Time allowed for learners to explain and for whole class discussion.

12. Assessment, formative and summative, based on the key objectives in the National Curriculum

The course is fully resourced with:

- Learner Practice Books, with detailed lesson plans for teachers
- Number cards, shape cards and number lines
- Interactive resources for reinforcement, revision and extension

Resources are freely available at: http://www.cimt.org.uk/projects/mepres/primary/

Mathematics Enhancement Programme

TEACHING SUPPORT: Book 4

LEARNING OBJECTIVES

At the end of Book 4 we expect learners to

- *use numbers up to 10 000 in calculations (addition, subtraction, multiplication and division) with confidence*
- *have instant recall of multiplication tables up to 10 × 10 and number bonds up to 10*
- *understand equivalent fractions and be able to add and subtract fractions*
- *understand the decimal equivalent of fractions with tenths and hundredths and be able to convert simple fractions to/from decimals*
- *add and subtract decimal numbers, using the column notation*
- *understand and solve simple equations*
- *express natural numbers in terms of their prime factors*
- *round numbers to the nearest 10, 100, 1000*
- *use Venn diagrams to classify a set of numbers*
- *extend units of measurement to include mm*
- *understand and use negative numbers on a number line and in context, e.g. thermometer, sea level*
- *order a set of numbers, including negative numbers*
- *understand simple calculations, additions and subtractions, with negative numbers*
- *find the perimeter, area and volume of simple shapes with lengths given as natural numbers, fractions or decimals*
- *understand the concepts of symmetry and congruence of simple 2D shapes and identify lines of symmetry or use mirror lines*
- *recognise and understand convex and concave shapes*
- *use positive 2D coordinates to define shapes*
- *use tally charts, pie charts, bar charts and pictograms to illustrate data*
- *understand what is meant by the median of a set of numbers*
- *find simple probabilities.*

Mathematics Enhancement Programme

TEACHING SUPPORT: Book 4

LEARNING OBJECTIVES with ILLUSTRATIVE EXAMPLES (and Answers appended)

At the end of Book 4 we expect learners to be confident with the following concepts.

Use numbers up to 10 000 in calculations (addition, subtraction, multiplication and division) with confidence.

Example 1 Practise calculation.

a)

		5	8	7
	5	3	4	2
+	7	7	9	3

b)

8	0	4	3	2
–	5	6	7	9

c)

	3	5	2	8
			×	5

d)

9	8	1	2	8	8

e)

	6	5	9	0
			×	8

f)

3	4	7	0	8
– 1	0	5	3	2

g)

1	4	0	3	5
			×	7

h)

4	1	0	1	0	4

i)

1	1	6	7	3	2

Have instant recall of multiplication tables up to 10 × 10 and number bonds up to 10.

Example 2 Complete the multiplication table. Make sure that you know it by heart.

×	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	0		0		0		0		0	0			0	
1	0	1	2	3		5		7	8		10			13
2	0	2	4			10	12			18			24	
3			6				18			27		33		
4			8	12			24	28		36	40			
5	0			15	20			35	40			55		
6		6			24	30			48			66		78
7	0		14	21			42	49		63	70		84	
8	0			24	32						80			104
9		9	18			45			72			99		
10	0			30			60	70						130
11	0		22					77			110			
12		12			48		72							
13	0			39		65				117				

Understand equivalent fractions and be able to add and subtract fractions.

Example 3 a) Join up the equal numbers.

$\frac{2}{5}$ $\frac{1}{2}$ $\frac{4}{3}$ $\frac{5}{2}$ $\frac{10}{20}$ $\frac{5}{10}$

2 $\frac{3}{6}$ $\frac{4}{10}$ $1\frac{1}{3}$ $\frac{6}{3}$ $\frac{4}{2}$

Example 4

a) $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \square$

b) $\frac{3}{8} + \frac{2}{8} = \square$

c) $\frac{7}{12} - \frac{2}{12} = \square$

d) $\frac{11}{20} - \square\frac{9}{20} = \square$

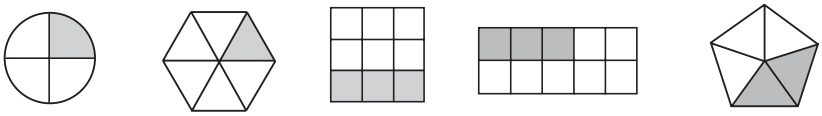
e) $\frac{7}{10} + \frac{3}{5} = \square$

f) $\frac{3}{4} - \frac{3}{8} = \square$

Understand the decimal equivalent of fractions with tenths and hundredths and be able to convert simple fractions to/from decimals.

Example 5 Join the numbers to the matching diagrams.

$\frac{3}{9}$ 0.3 $\frac{1}{4}$ $\frac{4}{10}$ 0.4 $\frac{2}{5}$



$\frac{2}{12}$ 0.25 $\frac{2}{8}$ $\frac{1}{3}$ $\frac{1}{6}$ $\frac{3}{10}$

Add and subtract decimal numbers, using the column notation.

Example 6

a)

		2	4
	1	0	3
+	8	7	2

b)

	4	2	1
		5	6
+		0	7

c)

	1	2	3	6
		1	7	2
+	4	9	5	8

d)
$$\begin{array}{r} 4\,9.6 \\ -\;1\,6.2 \\ \hline \end{array}$$

e)
$$\begin{array}{r} 8\,9.5 \\ -\;5\,2.6 \\ \hline \end{array}$$

f)
$$\begin{array}{r} 4\,2.1\,5 \\ -\;\;8.9\;\; \\ \hline \end{array}$$

Understand and solve simple equations.

Example 7 Which numbers can be written instead of the letters?

a) $a + 3.4 = 5.6$ b) $b - \square 3.1 = 0$ c) $c + 2.7 = 10$

$a =$ ☐ $b =$ ☐ $c =$ ☐

Express natural numbers in terms of their prime factors.

Example 8 Factorise 1250 and 175.

What is: i) the greatest ii) the smallest

natural number which is a factor of both numbers? i) ☐ ii) ☐

Round numbers to the nearest 10, 100, 1000.

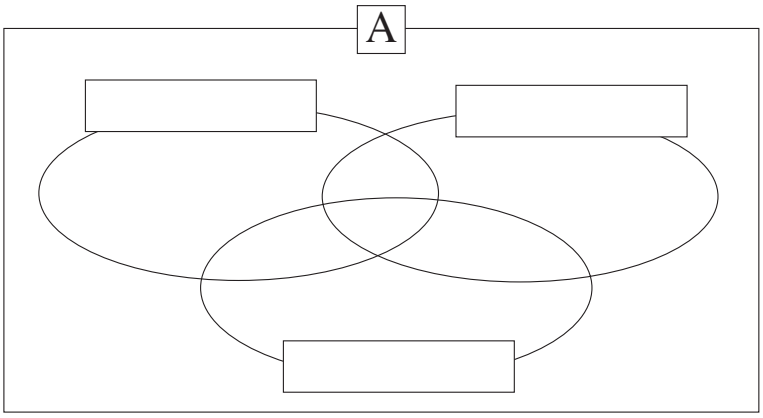
Example 9 Round the numbers.
Complete the table.

Number	Rounded to the nearest:		
	ten	hundred	thousand
4			
36			
50			
95			
172			
600			
999			
1050			
1846			

Use Venn diagrams to classify a set of numbers.

Example 10 How could you put these numbers into sets? Label each set, then write the numbers in the correct places.

Set A = {11, 7, 14, 23, 1, 25, 49, 70, 15, 45, 3, 100, 47, 19, 2}



Extend units of measurement to include mm.

Example 11 Change the units of measure, then round them to the nearest whole unit required.

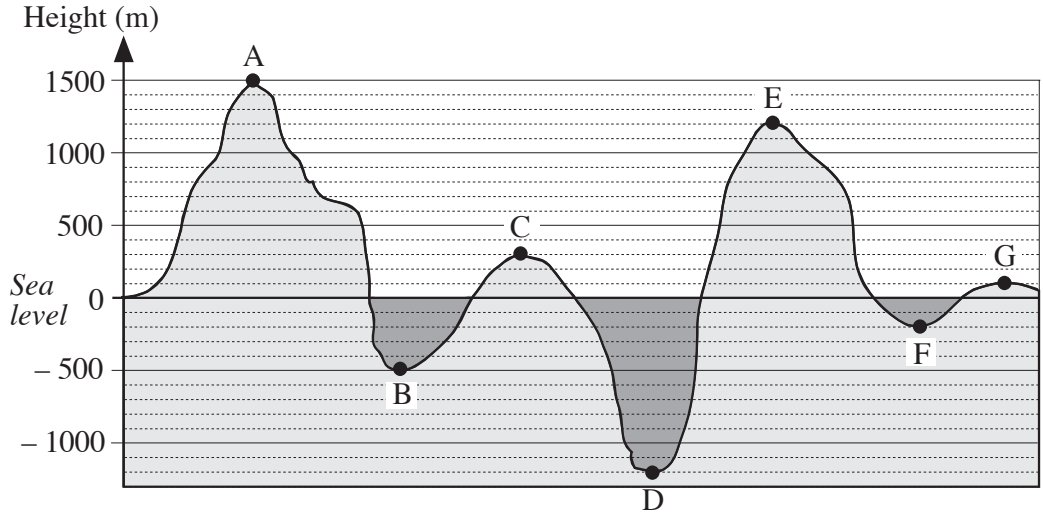
a) 678 m = ☐ km ☐ m ≈ ☐ km

b) 15 240 m = ☐ km ☐ m ≈ ☐ km

c) 5648 mm = ☐ m ☐ mm ≈ ☐ m

Understand and use negative numbers on a number line and in context, e.g. thermometer, sea level.

Example 12 Read the heights of the mountains and the depths of the bottom of the sea from this geographical cross-section and write them in the boxes. *Sea level* is 0 m.



A: ☐ m B: ☐ m C: ☐ m D: ☐ m

E: ☐ m F: ☐ m G: ☐ m

Write the heights in decreasing order.

. . . . . .

Order a set of numbers, including negative numbers.

Example 13 Write these temperatures in increasing order.

$-120°C,\ -31°C,\ -40°C,\ 0°C,\ -63°C,\ -2°C,\ -14°C,\ -6°C$

. . . . . .

Understand simple calculations, additions and subtractions, with negative numbers.

Example 14 Fill in the missing numbers. Check by drawing 1 and -1 for each part.

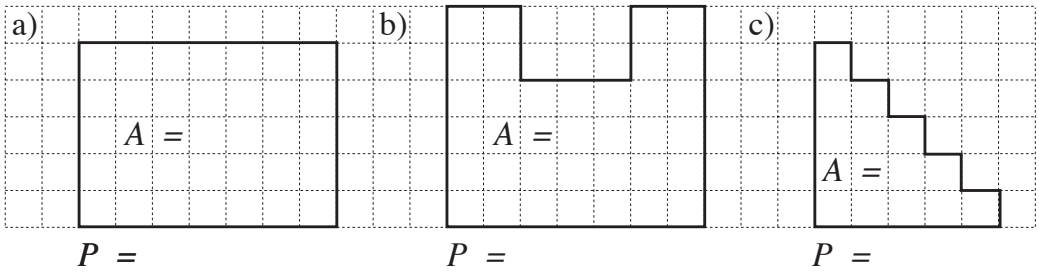
a) $5 + 2 =$ ☐ b) $5 + (-2) =$ ☐

c) $5 + (-8) =$ ☐ d) $-6 + 5 =$ ☐

e) $-6 + 6 =$ ☐ f) $-6 + (-1) =$ ☐

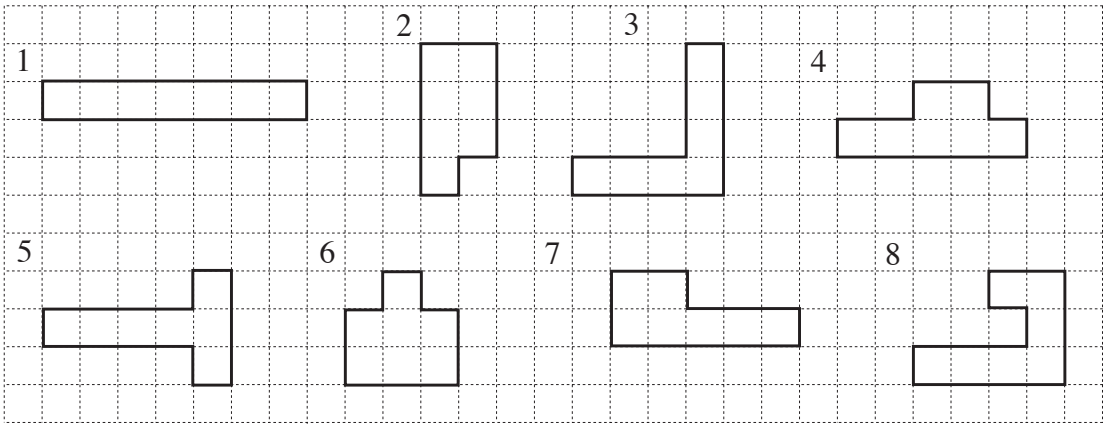
Find the perimeter, area and volume of simple shapes with lengths given as natural numbers, fractions or decimals.

Example 15 Measure, count or calculate the perimeter and area of the polygons.



Understand the concepts of symmetry and congruence of simple 2D shapes and identify lines of symmetry or use mirror lines.

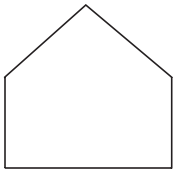
Example 16

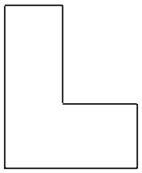


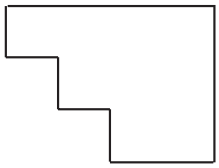
Colour the shapes which are **symmetrical** and draw the **lines of symmetry.**

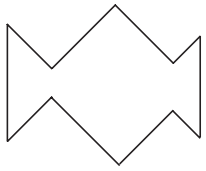
Recognise and understand convex and concave shapes.

Example 17 Draw over the sets of **parallel lines** in the same colour. Mark the **right angles.**

i) 

ii) 

iii) 

iv) 

Write C in the shapes which are **convex** and N in the shapes which are not convex.

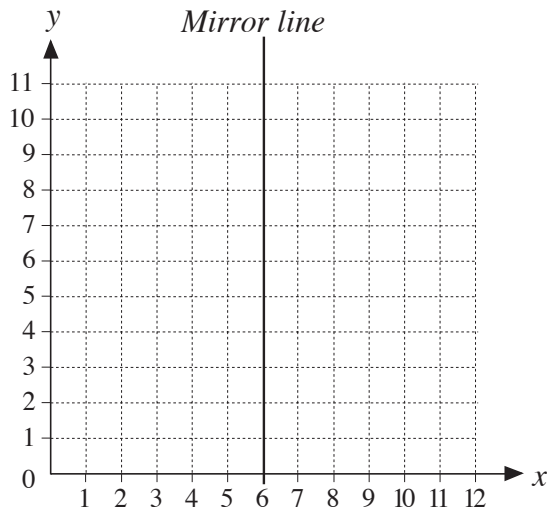
Use positive 2D coordinates to define shapes.

Example 18 Find these points on the grid and join them up.

(6, 1), (5, 4), (2, 2),

(4, 5), (1, 6), (4, 7),

(2, 10), (5, 8), (6, 11).



b) **Reflect** your shape in the *mirror line*.

c) How many vertices has the shape you have drawn? ☐

d) Is it convex or concave?

. . . . . .

e) What is its name? . . . . . .

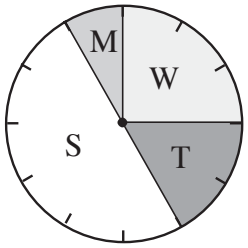
Use tally charts, pie charts, bar charts and pictograms to illustrate data.

Example 19 60 pupils were given a choice of 4 activities.

How many pupils chose each one and what fraction of them chose it? Use the **pie chart** to complete the table.

Activities

M: Museum
W: Walking
T: Theatre
S: Sports



	M	S	T	W
No. of pupils				
Fraction				

Example 20 a) How many acorns did the *Squirrel* family collect each day? Complete the diagram.

b) How many acorns did they collect altogether?

= 150 acorns

Monday:  $5 \times 150 =$ .....

Tuesday:  .....

Wednesday:  .....

Thursday:  .....

Friday:  .....

Saturday:  .....

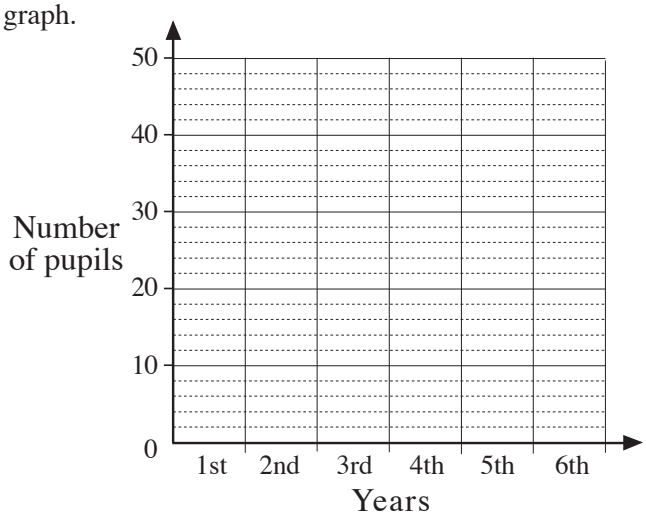
Sunday: .....

Understand what is meant by the median of a set of numbers.

Example 21 The table shows the number of pupils in the different years in a school.

Year	1st	2nd	3rd	4th	5th	6th
No. of pupils	42	40	46	42	38	41

a) Show the data in the graph.



b) Write the pupil numbers in increasing order.

. . . . . .

c) What is the **median**? . . . . . .

Find simple probabilities.

Example 22 A marble is dropped into this maze and has an equal chance of falling to the left or to the right.

a) In how many ways can the marble come out at:

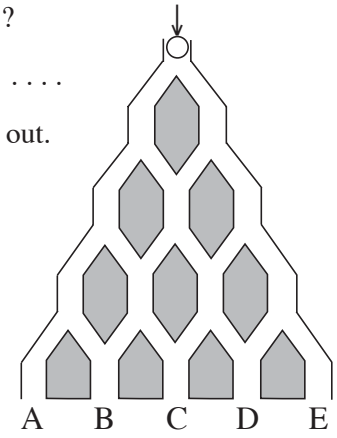
A . . . . B . . . . C . . . . D . . . . E . . . . ?

b) Where is the marble most likely to come out? . . . .

c) Write the ratio of the chance of where it comes out.

A B C D E

☐ : ☐ : ☐ : ☐ : ☐



LEARNING OBJECTIVES EXAMPLES: Answers

Example 1

a)

		5	8	7
	5	3	4	2
+	7	7	9	3
1	3	7	2	2

b)

8	0	4	3	2
–	5	6	7	9
7	4	7	5	3

c)

	3	5	2	8
			×	5
1	7	6	4	0

d)

		9	0	3	2
9	8	1	2	8	8

e)

	6	5	9	0
			×	8
5	2	7	2	0

f)

	3	4	7	0	8
–	1	0	5	3	2
	2	4	1	7	6

g)

1	4	0	3	5
			×	7
9	8	2	4	5

h)

		2	5	2	6	
4	1	0	1	0	4	

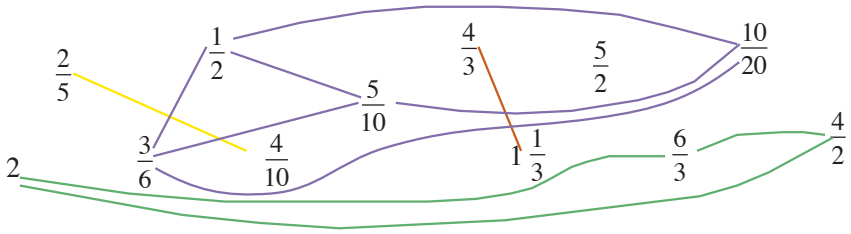
i)

			6	1	2
1	1	6	7	3	2

Example 2

×	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	0	**0**	0	**0**	0	**0**	0	**0**	0	0	**0**	**0**	0	**0**
1	0	1	2	3	**4**	5	**6**	7	8	**9**	10	**11**	**12**	13
2	0	2	4	**6**	**8**	10	12	**14**	**16**	18	**20**	**22**	24	**26**
3	**0**	**3**	6	**9**	**12**	**15**	18	**21**	**24**	27	**30**	33	**36**	**39**
4	**0**	**4**	8	12	**16**	**20**	24	28	**32**	36	40	**44**	**48**	**52**
5	0	**5**	**10**	15	20	**25**	**30**	35	40	**45**	**50**	55	**60**	**65**
6	**0**	6	**12**	**18**	24	30	**36**	**42**	48	**54**	**60**	66	**72**	78
7	0	**7**	14	21	**28**	**35**	42	49	**56**	63	70	**77**	84	**91**
8	0	**8**	**16**	24	32	**40**	**48**	**56**	**64**	**72**	80	**88**	**96**	104
9	**0**	9	18	**27**	**36**	45	**54**	**63**	72	**81**	**90**	99	**108**	**117**
10	0	**10**	**20**	30	**40**	**50**	60	70	**80**	**90**	**100**	**110**	**120**	130
11	0	**11**	22	**33**	**44**	**55**	**66**	77	**88**	**99**	110	**121**	**132**	**143**
12	**0**	12	**24**	**36**	48	**60**	72	**84**	**96**	**108**	**120**	**132**	**144**	**156**
13	0	**13**	**26**	39	**52**	65	**78**	**91**	**104**	117	**130**	**143**	**156**	**169**

Example 3

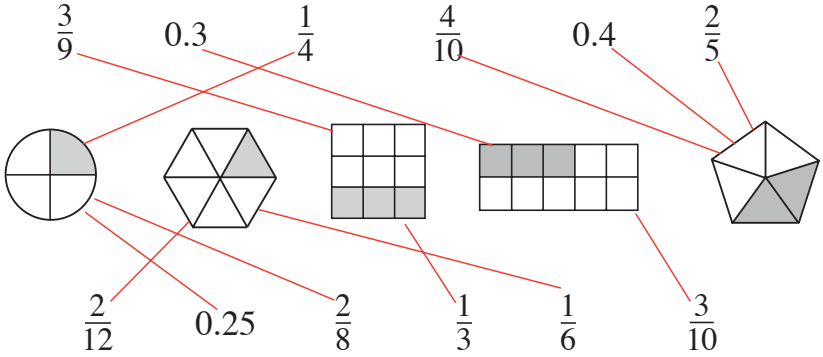


Example 4

a) $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\boxed{\frac{3}{5}}$ b) $\frac{3}{8}+\frac{2}{8}=\boxed{\frac{5}{8}}$ c) $\frac{7}{12}-\frac{2}{12}=\boxed{\frac{5}{12}}$

d) $\frac{11}{20}-\frac{9}{20}=\boxed{\frac{2}{20}=\frac{1}{10}}$ e) $\frac{7}{10}+\frac{3}{5}=\boxed{\frac{13}{10}=1\frac{3}{10}}$ f) $\frac{3}{4}-\frac{3}{8}=\boxed{\frac{3}{8}}$

Example 5



Example 6

a)

		2	4
	1	0	3
+	8	7	2
	9	9	9

b)

	4	2	1
		5	6
+		0	7
	4	8	4

c)

	1	2	3	6
		1	7	2
+	4	9	5	8
	6	3	6	6

d)

	4	9	6
−	1	6	2
	3	3	4

e)

	8	9	5
−	5	2	6
	3	6	9

f)

	4	2	1	5
−		8	9	
	3	3	2	5

Example 7

a) $a + 3.4 = 5.6$ b) $b - \square 3.1 = 0$ c) $c + 2.7 = 10$

$a =$ 2.2 $b =$ 3.1 $c =$ 7.3

Example 8

Factors of 1250

$2 \times 5 \times 5 \times 5 \times 5$

Factors of 175

$5 \times 5 \times 7$

i) 25

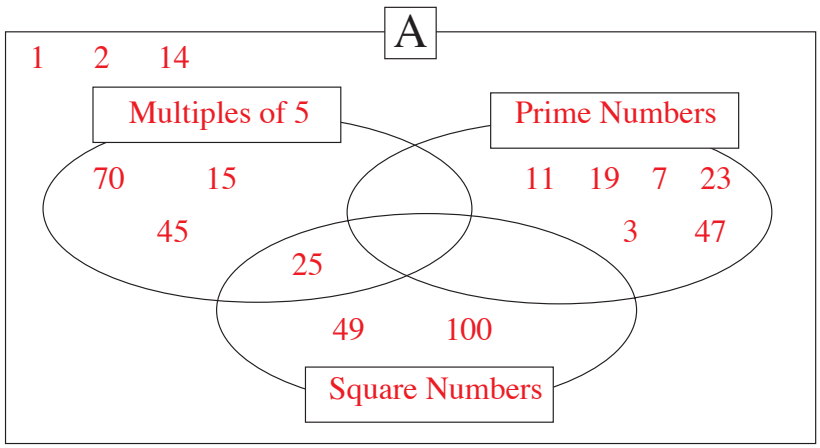
ii) 1

Example 9

Number	Rounded to the nearest:		
	ten	hundred	thousand
4	0	0	0
36	40	0	0
50	50	100	0
95	100	100	0
172	170	200	0
600	600	600	1000
999	1000	1000	1000
1050	1050	1100	1000
1846	1850	1800	2000

Example 10

e.g.



Example 11

a) 678 m = 0 km 678 m ≈ 1 km

b) 15 240 m = 15 km 240 m ≈ 15 km

c) 5648 mm = 5 m 648 mm ≈ 6 m

Example 12

A: 1500 m B: –500 m C: 300 m D: –1200 m

E: 1200 m F: –200 m G: 100 m

Heights in decreasing order:

1500 m > 1200 m > 300 m > 100 m > –200 m > –500 m > –1200 m

Example 13

Temperatures in increasing order:

–120 °C < –63 °C < –40 °C < –31°C < –14 °C < –2 °C < –0.6 °C < 0 °C

Example 14

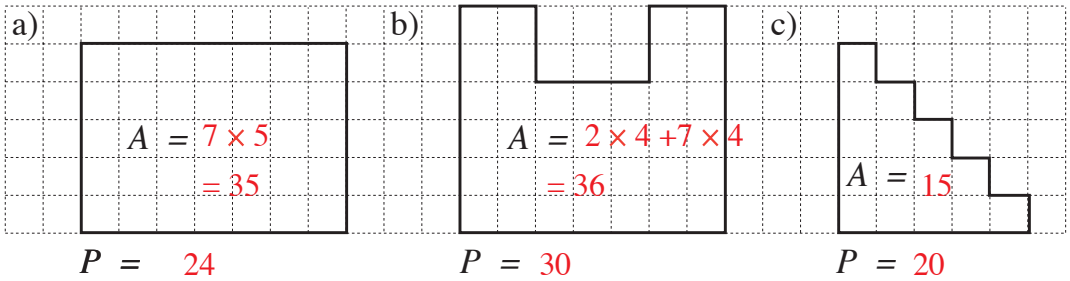
Fill in the missing numbers. Check by drawing 1 and – 1 for each part.

a) 5 + 2 = 7 b) 5 + (– 2) = 3

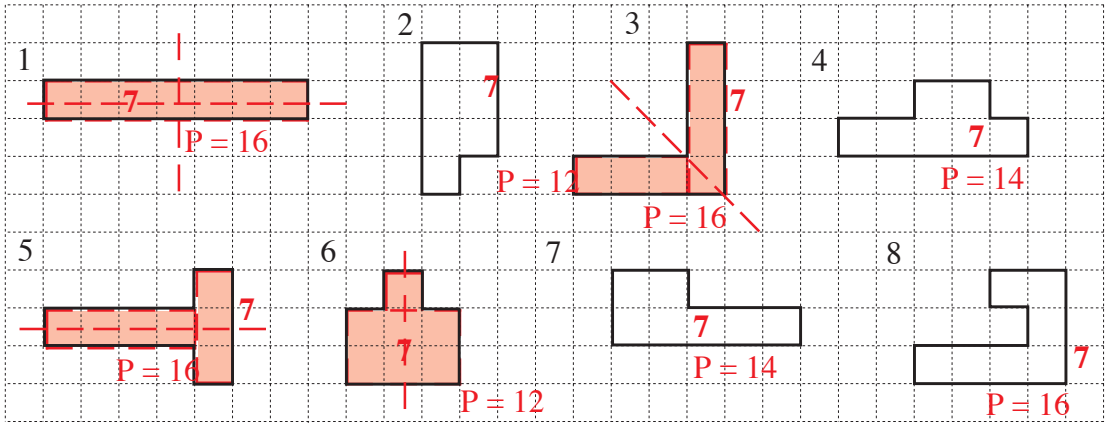
c) 5 + (– 8) = – 3 d) – 6 + 5 = –1

e) – 6 + 6 = 0 f) – 6 + (– 1) = – 7

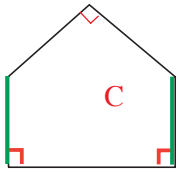
Example 15

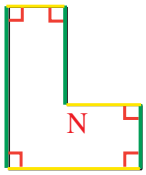


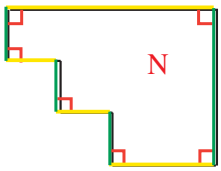
Example 16

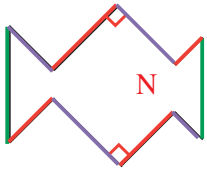


Example 17

i) 

ii) 

iii) 

iv) 

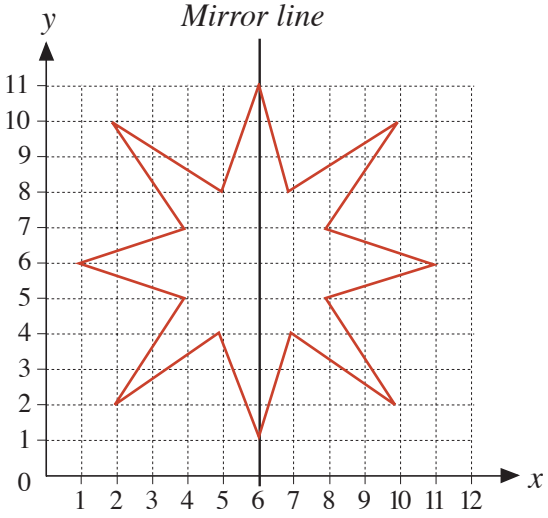
Example 18

a) b) diagram

c) 16

d) concave

e) 8-pointed star



Example 19

Count how many of the given units are in the perimeter and area of each shape.

	M	S	T	W
No. of pupils	5	30	10	15
Fraction	$\frac{1}{12}$	$\frac{6}{12}=\frac{1}{2}$	$\frac{2}{12}=\frac{1}{6}$	$\frac{3}{12}=\frac{1}{4}$

Example 20

Reflect the shape in one **axis** first. Then reflect the shape and its mirror image in the other **axis**. Draw the mirror lines of the whole shape.

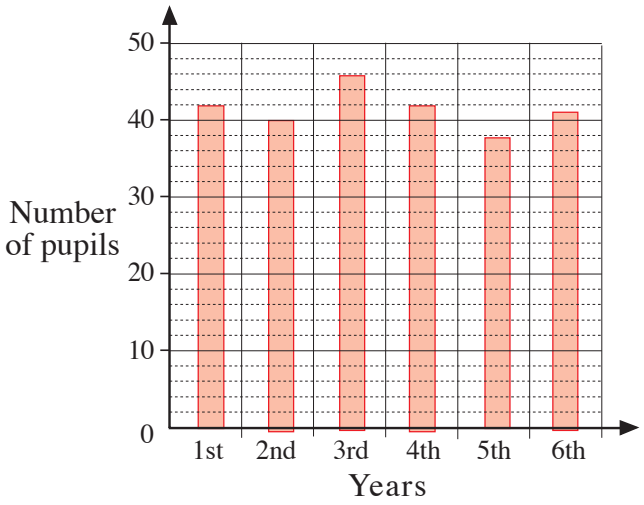
Monday	750	750
Tuesday	$4 \times 150 =$	600
Wednesday	$5 \times 150 + 75 =$	825
Thursday	$3 \times 150 =$	450
Friday	$4 \times 150 + 75 =$	675
Saturday	$3 \times 150 + 75 =$	525
Sunday	0	0
	Altogether:	3825

Example 21

a) graph

b) 38, 40, 41, 42, 42, 46

c) $\frac{41 + 42}{2} = 41.5$



c) What is the volume of each of the 6 cuboids? Write it beside them.

Example 22

a) A . . 1 . . B . . 4 . . C . . 6 . . D . . 4 . . E . . 1 . .

b) C

c) A B C D E

1 : 4 : 6 : 4 : 1

Mathematics Enhancement Programme: BOOK 4

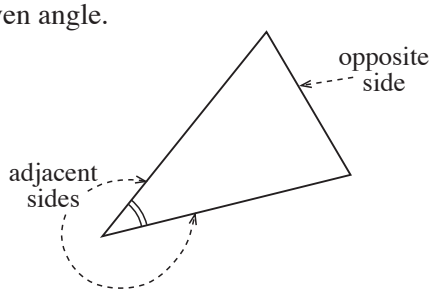
KEY FACTS and GLOSSARY

Adjacent and opposite sides of a triangle
Area
Average
Calendar
Capacity
Compass points
Concave and Convex shapes
Congruent shapes
Counting numbers
Decimal numbers
Direction
Edge
Enlarge/Reduce
Equals
Even/Odd
Face
Factors and multiples
Flow chart
Fractions: adding/subtracting
Fractions: equivalent
Illustrating data
Increasing/decreasing sequences
Inequalities
Integers (or whole numbers)
Magic square
Mirror lines and symmetry
Multiplication tables
Nets
Number bonds
Number lines
Parallel and perpendicular lines
Perimeter
Product
Quotient and remainder
Random
Range
Reduce: see Enlarge/reduce
Roman numerals
Rounding
Sequence
Shapes: plane (2-dimensional)
solid (3 dimensional)
Similarity
Subtrahend
Tally
Tree diagram
Units of measurement
Venn diagram
Vertex
Volume

Underlined topics have been added to the BOOK 3 Key Facts and Glossary

Adjacent and opposite sides

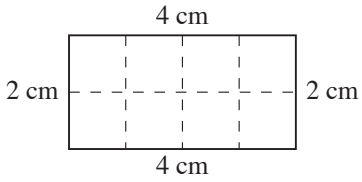
Adjacent sides are the two sides that meet at a given angle. The *opposite* side is the side opposite the given angle.



For the angle marked in this triangle the two *adjacent* sides are marked; the third side is the *opposite* side (opposite the given angle).

Area

The area of a plane shape is a measurement of the surface area enclosed by the shape.



The shape shown has an area of 8 square cm.

Average

This the number that best represents a set of numbers. Usually it is the *mean* but it could be the *median* (middle value) or the *mode* (most frequent value).

For example, for the set $\{1, 2, 2, 5, 6, 6, 6\}$, the

$$\text{mean} = (1 + 2 + 2 + 5 + 6 + 6 + 6) \div 7 = 28 \div 7 = 4$$

$$\text{median} = 5$$

$$\text{mode} = 6$$

Calendar

There are 7 DAYS in a WEEK:

Monday	Friday
Tuesday	Saturday
Wednesday	Sunday
Thursday	

and 28, 29, 30 or 31 DAYS in a MONTH:

January	31 days
February	28 (29 in a Leap Year) days
March	31 days
April	30 days
May	31 days
June	30 days
July	31 days
August	31 days
September	30 days
October	31 days
November	30 days
December	31 days

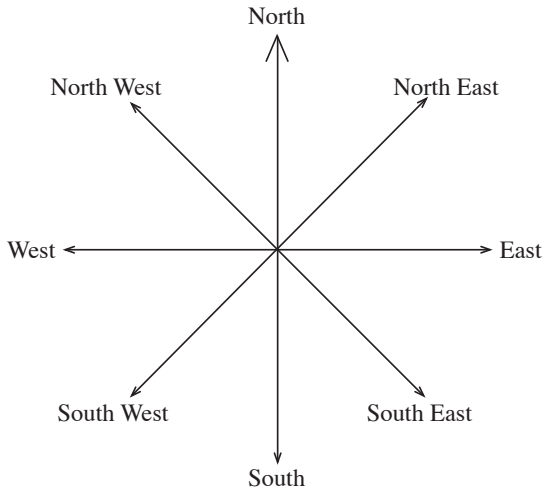
There are 4 SEASONS in a YEAR: Spring, Summer, Autumn, Winter

Capacity

This is the maximum quantity that a container can hold.

For example, milk is often sold in 1, 2 or 4 litre plastic containers.

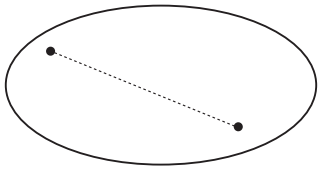
Compass points



Concave and convex shapes

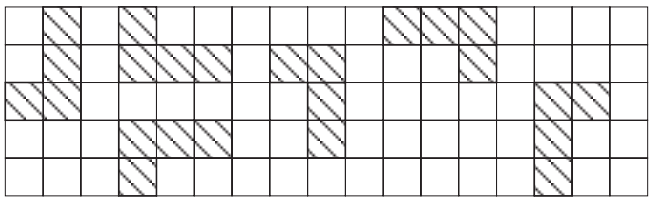
Concave If a straight line that is inside the shape cannot always be drawn between any two points on the shape, the shape is concave. The two points are *inside* the shape but the straight line drawn between them passes *outside* the shape.

Convex If straight line drawn between any two points on the shape will always lie *inside* the shape, as can be seen from the example below.



Congruent shapes

Shapes that are identical in size and shape are said to be *congruent*.



Six congruent shapes are shown in this diagram.

Counting numbers

This is the set of numbers 1, 2, 3, 4, 5, 6, ...

These are also called the *natural numbers* (or *positive integers*)

Decimal number

A number that includes a decimal point followed by digit(s) that show a value smaller than one.

For example, 76.8 is a decimal number.

$$76.8 = 76\frac{8}{10}$$

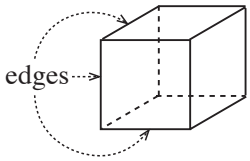
Generalising, $0.a = \frac{a}{10}$ $\quad a = 0, 1, \ldots, 9()$

$0.ab = \frac{a}{10} + \frac{b}{100}$ $\quad a,b = 0, 1, 2, \ldots, 9()$

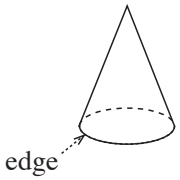
Direction left and right; above and below; in front and behind

Edge

The edge of a shape is the line at which two surfaces of the shape meet.



This shape has 12 edges.



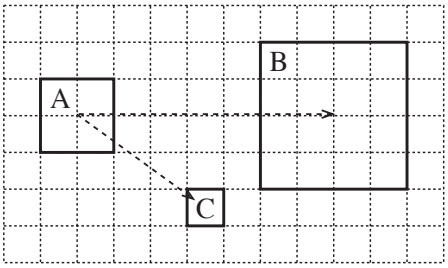
This shape has 1 edge.

Enlarge/Reduce

Shapes can be enlarged (made larger) or reduced (made smaller) by a scale factor.

For example, in the diagram below, shape A has been enlarged by scale factor 2 to give shape B;

shape A has been reduced by scale factor $\frac{1}{2}$.



Equals

Equals means 'the same as'. We use the sign =

For example, $2 + 2 = 3 + 1\,(= 4)$

Even/Odd

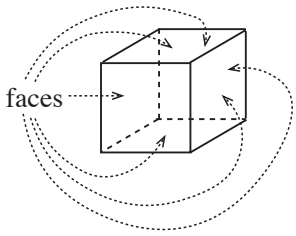
2, 4, 6, 8, 10, 12, 14, 16, 18, ... are EVEN numbers (they can each be divided exactly by 2)

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, ... are ODD numbers (they cannot be divided exactly by 2)

Face

The side of a solid shape is called a face.

The shape shown here has 6 faces.



Factors and Multiples

Any whole number that divides exactly into a whole number with no remainder is called a *divisor* or *factor* of the number.

For example, 1, 2, 3, 4, 6 and 12 are all divisors (or factors) of 12.

Any whole number that can be divided by a whole number with no remainder is called a *multiple* of the number.

For example, 5, 10, 15, 20, . . . are all multiples of 5.

Flow chart

A diagrammatic set of instructions to follow to produce an outcome.

Fractions: adding/subtracting

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

(a, b and c are natural numbers, that is, numbers used for counting)

$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

Fractions: equivalent

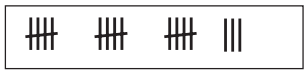
$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \ldots$$

$$\frac{1}{10} = \frac{5}{50} = \frac{10}{100} = \ldots$$

Illustrating data

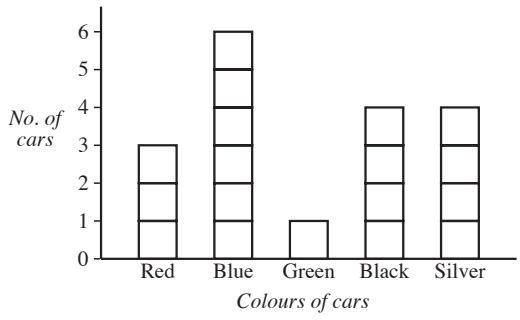
Three of the ways of illustrating data are with a:

Tally Chart



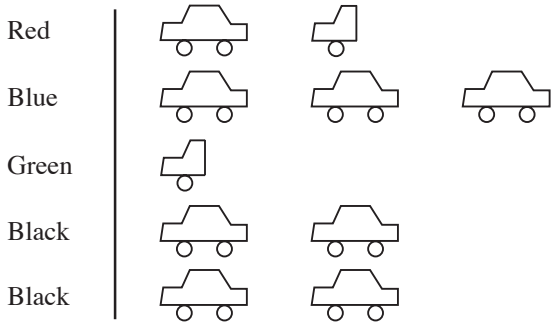
This tally chart represents 18 items of data.

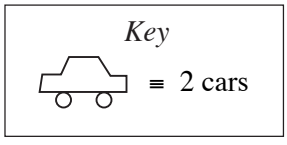
Bar Chart



This bar chart represents 18 items of data (3 Red, 6 Blue, 1 Green, 4 Black and 4 Silver cars)

Pictogram





This pictogram represents the 18 cars above.

A pictogram must always have a key.

Increasing/decreasing sequences

1, 4, 7, 10, 13, 16, ... is an increasing sequence with the rule 'add 3' $(+3)$

15, 13, 11, 9, 7, ... is a decreasing sequence with the rule 'subtract 2' (-2)

Inequalities

We use *inequality signs* when we compare two numbers: that is,

5 is greater than 3 is written as $5 > 3$

2 is less than 4 is written as $2 < 4$

So the symbol '>' means 'is greater than'

and '<' means 'is less than'.

The symbol '$\geq$' means 'is less than or is equal to'.

For example, if $\square \geq 4$, then $\square$ can be 4, 5, 6, ... (assuming that $\square$ is a counting number).

Similarly, '$\leq$' means 'is more than or is equal to'.

For example, if $\square \leq 6$, then $\square$ can be 6, 5, 4, 3, 2 or 1 (assuming that $\square$ is a counting number).

A small number *inside* an inequality sign shows how much greater or smaller the second number is than the first number.

For example, $3 <^{2} 5$ means that 3 is 2 less than 5

and $7 \ ^{5}> 2$ means that 7 is 5 greater than 2.

Integers (or whole numbers)

This is the set of whole numbers, both positive and negative.

For example, $..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...$

Magic square

In a magic square, each of the rows, columns and diagonals have the same total.

For example,

8	3	4
1	5	9
6	7	2

The magic number in this magic square is 15.

Minuend

In a subtraction, the first number in a subtraction is the minuend;

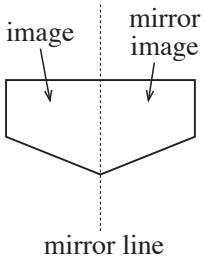
minuend – subtrahend = difference

For example, $47 - 23 = 24$

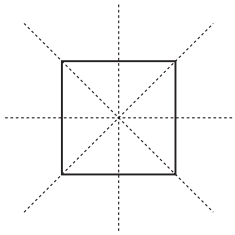
↑ minuend ↑ subtrahend ↑ difference

Mirror lines and symmetry

The whole shape has one line of symmetry.



Four lines of symmetry are shown here.



Multiplication tables

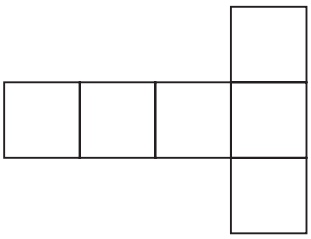
Illustrated here for numbers up to 10×10.

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Nets

A net is a 2 dimensional representation of a 3D shape.

For example,



This net folds to form a cube.

Number bonds

The term 'number bonds' usually refers to the addition of two single-digit positive numbers,

For example, number bonds adding to 12 are

$0 + 12 = 12$, $1 + 11 = 12$, $2 + 10 = 12$, $3 + 9 = 12$, $4 + 8 = 12$,

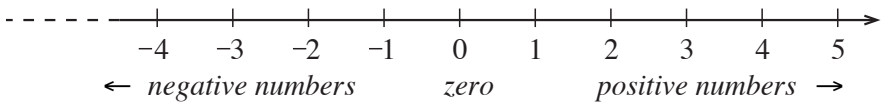
$5 + 7 = 12$, $6 + 6 = 12$, $7 + 5 = 12$, $8 + 4 = 12$, $9 + 3 = 12$,

$10 + 2 = 12$, $11 + 1 = 12$, $12 + 0 = 12$

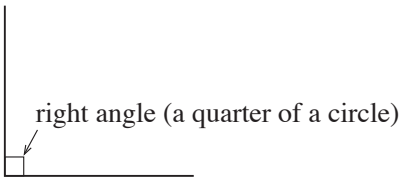
Number lines

Numbers, positive and negative, can be easily represented on a number line.

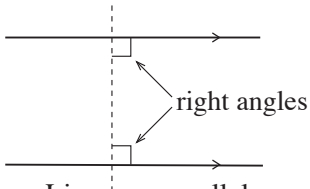
For example,



Parallel and perpendicular lines



Lines are perpendicular

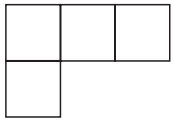


Lines are parallel

Perimeter

The perimeter of a plane shape is the total length of the line around its edge.

If the side length of each square is 1 unit, this shape has a perimeter of 10 units



Product

The product is the result of multiplying two numbers.

For example, 6 is the product of 2 and 3 and so we write $2 \times 3 = 6$

Quotient and remainder

When dividing one number by another, the result is the *quotient*.

For example, $6 \div 2 = 3$ 6 is exactly divisible by 2 and so 3 is the quotient.

$7 \div 2$ gives a quotient of 3 with remainder 1, so $7 \div 2 = 3$ remainder 1

Similarly, $11 \div 3 = 3$ (quotient) with remainder 2

Random

This means without pattern or predictability. A *random number* is a number chosen as if by chance from a specified set of numbers, with all the numbers having the same chance of being chosen.

For example, when throwing a dice each of the numbers 1, 2, 3, 4, 5 and 6 have the same chance of being thrown.

Range

The range of a set of numbers is the difference between the largest and the smallest numbers.

For example, for the set of numbers

$\{5, 6, 8, 10\}$, the range is $10 - 5 = 5$

Reduce see *Enlarge/Reduce*

Roman numerals

1	I		11	XI
2	II		12	XII
3	III		13	XIII
4	IV	(this means 1 before 5)	14	XIV
5	V		15	XV
6	VI		16	XVI
7	VII		17	XVII
8	VIII		18	XVIII
9	IX	(this means 1 before 10)	19	XIX
10	X		20	XX

This continues in the same way, with 50 represented by the letter L and 100 by C.

Rounding

Rounding means making a number easier to calculate with, changing its value as little as possible. It is used to approximate an answer.

For example, rounding to the nearest 10, the numbers

1, 2, 3 and 4 round down to 0

5, 6, 7, 8 and 9 round up to 10

11, 12, 13 and 14 round down to 10

15, 16, 17, 18 and 19 round up to 20

174 to the nearest 10 is 170

174 to the nearest 100 is 200.

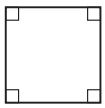
Note that, by convention, 5, 15, 25, . . . round up to 10, 20 and 30, respectively.

Sequence

This is a set of numbers (or objects) written order according to a rule.

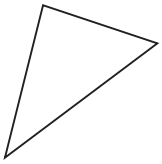
For example,	5, 10, 15, 20, 25, ...	(with 5 as the first number, increase by 5 each time to get the next term)
	2, 3, 5, 8, 12, 17, ...	(with 2 as the first number, add 1 and then add 1 more to the difference each time to get the next term)
	2, 4, 8, 16, 32, ...	(with 2 as the first number, multiply each number by 2 to get the next term)

Shapes: plane (2-dimensional)

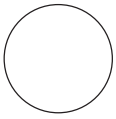


Square (all sides equal and four right angles)

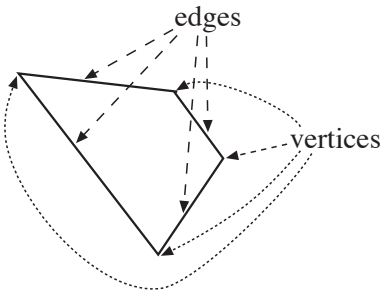
Rectangle (opposite sides equal and parallel and four right angles)



Triangle (3 straight sides)

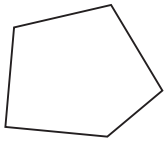


Circle

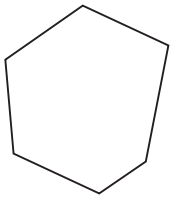


Quadrilateral - any shape with 4 straight sides (note that the sides are called *edges* and meet at *vertices).*

(Note that all squares are rectangles and all rectangles are quadrilaterals.)

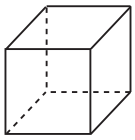


Pentagon - any shape with 5 straight edges (sides)

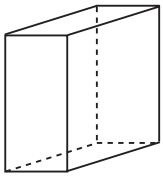


Hexagon - any shape with 6 straight edges (vertices)

Shapes: solid (3-dimensional)

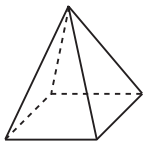


Cube (all sides equal so each face is a square)

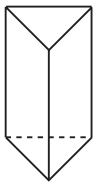


Cuboid (all opposite sides equal so each face is a rectangle)

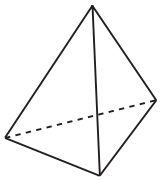
Sphere



Square-based pyramid



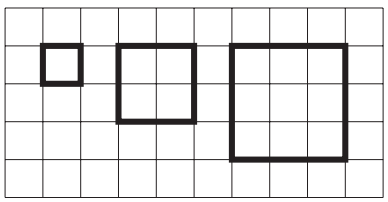
Triangle-based prism



Triangle-based pyramid

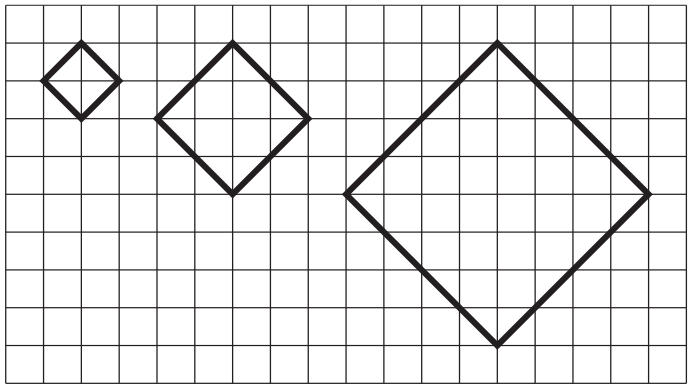
Similarity

(a)



These shapes are similar.

(b)



These shapes are similar.

(The sides are in the same ratio, that is, 1 : 1 in (a) and 1 : 2 (i.e, 2 : 4 and 3 : 6) in (b).)

Subtrahend

In a subtraction, the number that is subtracted from the minuend,

minuend – subtrahend = difference

For example, 47 – 23 = 24

↑ ↑ ↑

minuend subtrahend difference

Tally

This is a record of the frequency of an event; in an experiment we often dra 4 short vertical lines crossed by the fifth line to denote 5 occurences of the event.

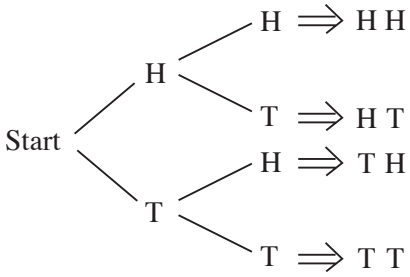
𝍸 𝍸 | ⇒ 11

𝍸 𝍸 𝍸 ||| ⇒ 18

Tree diagram

This is a diagram with branches used to show various outcomes and display all possible results when several events are combined.

For example, when tossing 2 coins and noting heads (H) or tails (T),



Units of measurement

Length:	10 mm	=	1 cm
	1000 mm	=	1 m
	100 cm	=	1 m
	1000 m	=	1 km

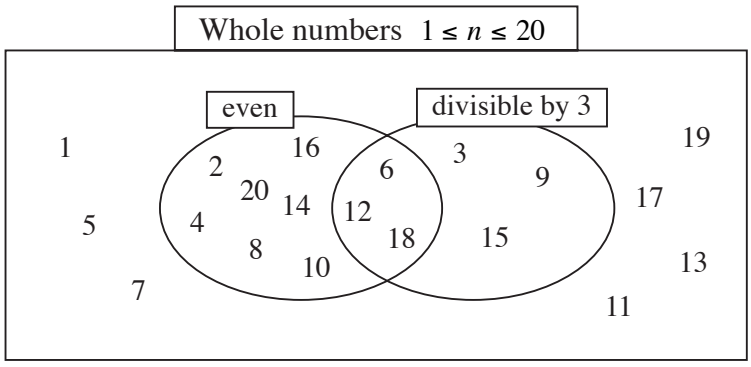
- - - - - - - - - - - - - - - - - - - -

Capacity:	10 ml	=	1 cl
	1000 ml	=	1 litre
	100 cl	=	1 litre
Mass:	1000 g	=	1 kg
	1000 kg	=	1 tonne
Time:	60 seconds	=	1 minute
	60 minutes	=	1 hour
	24 hours	=	1 day
	7 days	=	1 week
	52 weeks	=	1 year
	12 months	=	1 year

Venn diagram

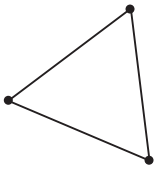
A diagram representing logical sets as circles or closed curves within an enclosing rectangle (the universal set). Common elements of the sets are represented by intersections of the circles.

For example, if set A is 'even numbers' and set B is 'numbers divisible by 3', then for whole numbers $0 \le n \le 20$, we can show the two sets as in the Venn diagram below.

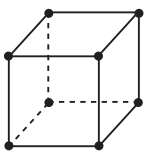


Vertex

The point at which two edges meet.

2D: 

There are 3 vertices on this triangle

3D: 

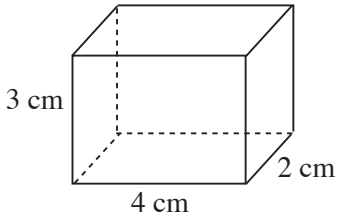
There are 8 vertices on this cuboid

Volume

The volume is the number of cubic units that will exactly fill a 3D shape.

For example, for this cuboid,

volume = 3 × 2 4 = 24 cubic cm



PROBLEM SOLVING

1. Write the numbers which have:

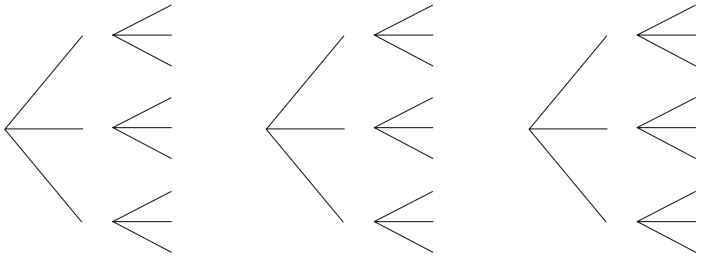
a) an even digit as their hundreds digit and 500 as their nearest ten.

b) an odd digit as their hundreds digit and 500 as their nearest ten.

c) the smallest even digit as their tens digit and 1010 as their nearest ten.

2. How many 3-digit numbers can you make from these digits? 5 6 1

a) Complete the tree diagrams.



b) List the numbers. ☐☐☐

3. *I thought of a number, then added 900.*
The result was a number less than 1000.

Write ✓ if you think the statement is true and **X** if you think it is false.

a) The number I first thought of must be less than 100. ☐

b) The number I first thought of must be less than 99. ☐

c) The number I first thought of could be equal to 99. ☐

d) The number I first thought of cannot be more than 99. ☐

e) The number I first thought of could be equal to 10. ☐

f) The number I first thought of cannot be 100. ☐

4. Estimate the product first, then do the multiplication.

a) *E:* ☐☐☐ 73×6 *E:* ☐☐☐ 146×3 *E:* ☐☐☐ 246×3 *E:* ☐☐☐☐ 346×3

b) *E:* ☐☐☐ 47×8 *E:* ☐☐☐ 147×3 *E:* ☐☐☐ 147×6 *E:* ☐☐☐ 247×3

5. Write the whole numbers up to 1000 which have 4 as the sum of their digits.

6. Are the statements true or false? Write T for true and F for false in each box.

a) Every number which is a whole hundred is divisible by 2. ☐

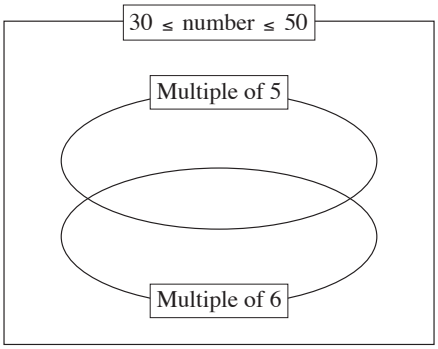
b) There is an even number which has 5 as its units digit. ☐

c) Every number which is divisible by 5 is a whole ten. ☐

d) 217 is divisible by neither 5 nor 2. ☐

e) Every number which is a whole ten is divisible by 2 and by 5. ☐

7. Write the whole numbers from 30 to 50 in the correct set.



8. Write in the boxes the numbers described.

a) The smallest 4-digit: i) number ☐ ii) odd number ☐

b) The greatest 4-digit: i) number ☐ ii) odd number ☐

c) The greatest 4-digit number divisible by: i) 5 ☐ ii) 10 ☐

d) The greatest 4-digit number divisible by 100 which has the same digit in its hundreds and thousands columns.

9. Estimate quickly, then calculate the sum.

a) 2653 + 1746

E: ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐ *C:* ☐☐☐☐☐

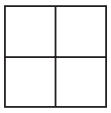
b) 1256 + 7902

E: ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐ *C:* ☐☐☐☐☐

c) 5343 + 2145

E: ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐ *C:* ☐☐☐☐☐

10. a) How many rectangles are in the diagram below? . . . . . .



b) How many rectangles would be in 874 such diagrams? . . . . . .

c) What is the **area** of the diagram? A = ...

d) What is the **perimeter** of the diagram?

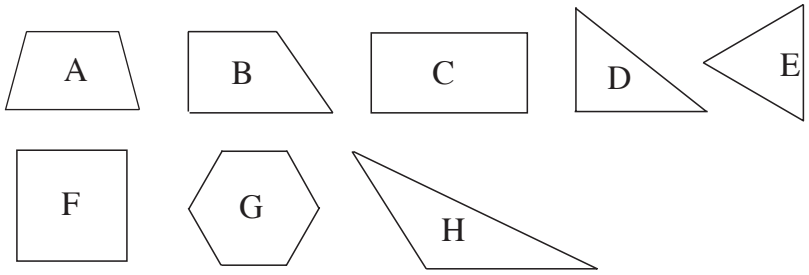
P = ...

11. a) In each diagram, mark

- right angles in *red* like this, 

- angles **smaller** than a right angle in *blue* like this, 

- angles **larger** than a right angle in *green* like this, 



b) List the letters of the shapes for which each statement is true.

i) It is a square. . . . . .

ii) It is a rectangle. . . . . .

iii) It is a quadrilateral. . . . . .

iv) It is a triangle. . . . . .

v) It has at least one right angle. . . . . .

vi) Every angle is a right angle. . . . . .

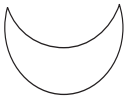
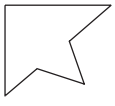
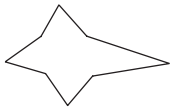
vii) It has at least one angle smaller than a right angle. . . . . .

viii) All its angles are smaller than a right angle. . . . . .

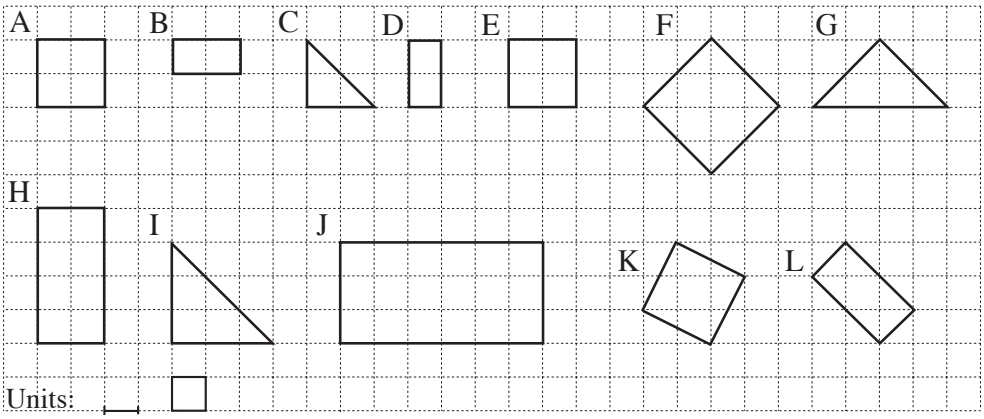
ix) It has at least one angle larger than a right angle. . . . . .

x) All its angles are larger than a right angle. . . . . .

12. Complete these non-convex shapes so that they become **convex** shapes.

a)  b)  c)  d)  e) 

13.



List the **similar** shapes.

Write the **area** inside each shape and the length of the **perimeter** below.

14. Complete the fractions.

a) $\frac{1}{2} = \frac{\square}{4} = \frac{4}{\square} = \frac{\square}{6} = \frac{\square}{10} = \frac{10}{\square} = \frac{\square}{100} = \frac{\square}{\square}$

b) $\frac{1}{4} = \frac{\square}{16} = \frac{2}{\square} = \frac{\square}{20} = \frac{8}{\square} = \frac{25}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

c) $\frac{1}{3} = \frac{2}{\square} = \frac{\square}{12} = \frac{3}{\square} = \frac{\square}{15} = \frac{\square}{24} = \frac{\square}{30} = \frac{100}{\square} = \frac{\square}{\square}$

15. a) $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \square$ b) $\frac{3}{8} + \frac{2}{8} = \square$ c) $\frac{7}{12} - \frac{2}{12} = \square$

d) $\frac{11}{20} - \frac{9}{20} = \square$ e) $\frac{7}{10} + \frac{3}{5} = \square$ f) $\frac{3}{4} - \frac{3}{8} = \square$

16. Compare the pairs of numbers and fill in the missing signs. (<, >, =)

Use the diagrams to help you.

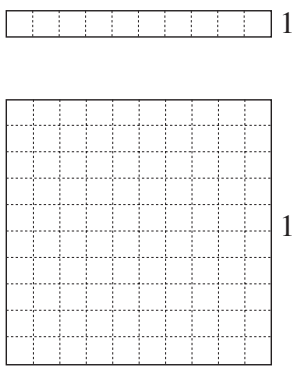
a) $\frac{2}{10} \square \frac{7}{10}$ $\frac{8}{10} \square 0.9$ $0.6 \square 0.3$

b) $\frac{15}{100} \square \frac{72}{100}$ $\frac{43}{100} \square 0.70$ $0.52 \square 0.49$

c) $0.04 \square 0.1$ $\frac{2}{10} \square \frac{18}{100}$ $0.27 \square 0.3$

d) $\frac{1}{5} \square 0.2$ $\frac{2}{5} \square 0.3$ $\frac{3}{10} \square 0.6$

e) $\frac{1}{5} \square \frac{17}{100}$ $\frac{3}{10} \square 0.51$ $\frac{78}{100} \square 0.53$



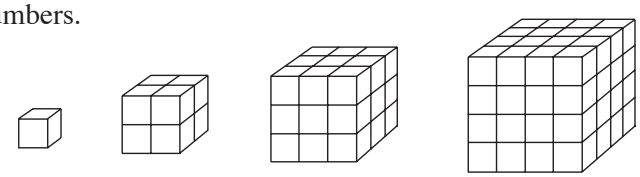
17. Which quantity is greater? Fill in the missing signs.

a) $\frac{3}{10}$ m ☐ 54 cm b) 0.9 kg ☐ 90 g c) $\frac{1}{6}$ hour ☐ 30 min

d) £150 20 p ☐ £150.2 e) $5\frac{7}{100}$ litres ☐ 5 litres 700 ml

f) $4\frac{1}{2}$ weeks ☐ 29 days g) 84.3 cm ☐ 843 mm ☐ 8.43 m

18. Imagine these cubes built from unit cubes.
Fill in the missing numbers.



Length of 1 edge ⟼	1	2	3	4	5	6
Area of cube ☐						
Volume of cube						

19. Follow the example. Complete the sentences. Use the number line to help you.

a) 8°C is greater than 3°C by 5°C. $8-3=5,\quad 5+3=8$

b) 3°C is ☐ than 8°C by 5°C. $3-8=$ ☐, ☐ $+8=3$

c) 8°C is greater than 0°C by ☐. $8-0=$ ☐, ☐ $+0=8$

d) 3°C is greater than −2°C by ☐. $3-(-2)=$ ☐, ☐ $+(-2)=3$

e) −2°C is less than 3°C by ☐. $-2-3=$ ☐, ☐ $+3=-2$

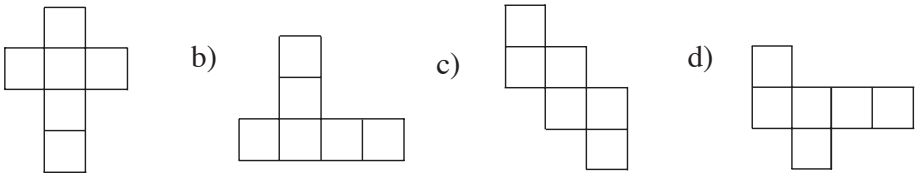
f) −2°C is ☐ than −5°C by 3°C. $-2-(-5)=$ ☐, ☐ $+(-5)=-2$

20. In an opaque bag there are 10 *black* and 30 *white* marbles.

What is the smallest number of marbles you must take out of the bag (with your eyes closed) to be **certain** of getting 2 marbles which are the same colour?

21. Circle the nets which can make a cube. Colour their opposite faces in the same colour.

a) b) c) d)



22. What is the smallest natural multiple of 2, 3, 4, 5 and 8?

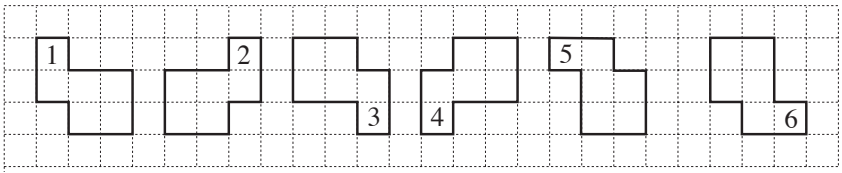
23. In an opaque bag, there are 5 *black*, 10 *red* and 5 *white* marbles.
What is the smallest number of marbles you must take out of the bag (with your eyes closed) to be **certain** of getting:

a) 3 marbles which are the same colour

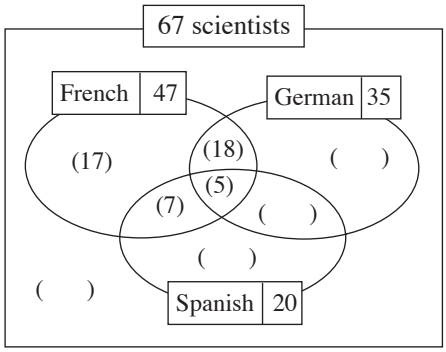
b) a *red* marble?

24. List in your exercise book all the numbers between 999 and 10 000 which have 4 as the sum of their digits. How many did you find?

25. These shapes are **congruent**. What has been done to *Shape 1* to make *Shape 2*, *Shape 2* to make *Shape 3*, and so on? Write it in your exercise book.



26. Among 67 scientists at a conference,
47 speak French,
35 speak German,
20 speak Spanish,
12 speak French and Spanish,
11 speak German and Spanish,
5 speak all three languages.



a) Complete the *Venn* diagram.

b) How many scientists speak:

i) only French ☐ ii) only German ☐ iii) only Spanish? ☐

c) How many scientists speak Spanish and German but not French? ☐

d) How many scientists speak neither Spanish nor German nor French? ☐

27. Circle the natural numbers up to 100 which have only two factors.
(e.g. the only factors of **7** are 7 and 1)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

We call these numbers **prime numbers**. List them in increasing order.

28. The perimeter of a triangle is 10 cm and the length of each side is a whole cm.

Are these statements true or false? Write a ✓ if true and a **X** if false.

a) The triangle has only one side which is 1 cm long. ☐

b) The triangle could have only one side which is 2 cm long. ☐

c) The triangle has only one side which is 3 cm long. ☐

d) The triangle has only one side which is 5 cm long. ☐

PROBLEM SOLVING - SOLUTIONS

1. *Question and Solution*

Write the numbers which have:

a) an even digit as their hundreds digit and 500 as their nearest ten.

495, 496, 497, 498, 499

b) an odd digit as their hundreds digit and 500 as their nearest ten.

500, 501, 502, 503, 504

c) the smallest even digit as their tens digit and 1010 as their nearest ten.

1005, 1006, 1007, 1008, 1009

Notes

Note that for a) and b), numbers that round to 500 as their nearest 10 are

495, 496, 497, 498, 499, 500, 501, 502, 503 and 504

Hence for a), we need those numbers that have an even digit as their hundreds digit, namely

495, 496, 497, 498, 499

whilst for b), we need those numbers where the hundreds digit is an odd number,

500, 501, 502, 503, 504.

For part c), numbers that round to 1010 as their nearest 10 are

1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014

and the smallest even digit is 0, so the answer needed is

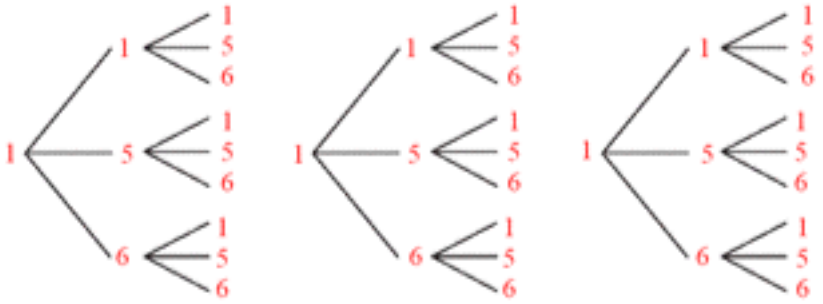
1005, 1006, 1007, 1008, 1009.

---

2. *Question and Solution*

How many 3-digit numbers can you make from these digits? 5 6 1

a) Complete the tree diagrams.



b) List the numbers. 111, 115, 116, 151, 155, 156, 161, 165, 166, 511, 515, 516, 551, 555, 556, 561, 565, 566, 611, 615, 616, 651, 655, 656, 661, 665, 666

Notes

The systematic method is developed by using the tree diagram approach. In the solution shown, note that the numbers are taken in the order 1, 5, 6 throughout. This ensures that the numbers developed, that is,

$$111,\ 115,\ 116,\ 151,\ \ldots,$$

are listed in increasing order (but any order is acceptable).

---

3. *Question and Solution*

I thought of a number, then added 900.

The result was a number less than 1000.

Write ✓ if you think the statement is true and **X** if you think it is false.

a) The number I first thought of must be less than 100. ✓

b) The number I first thought of must be less than 99. X

c) The number I first thought of could be equal to 99. ✓

d) The number I first thought of cannot be more than 99. ✓

e) The number I first thought of could be equal to 10. ✓

f) The number I first thought of cannot be 100. ✓

Notes

Note that, if we call the number thought of x, then

$$x + 900 < 1000$$

Now take 900 from each side of the equation, to give

$$x + 900 - 900 < 1000 - 900$$

$$x + 0 < 100$$

$$\boxed{x < 100}$$

Hence a) is true but, as x could be equal to 99, b) is false and c) is true. Similarly, d) is true and x could be

$$0,\ 1,\ 2,\ \ldots,\ 99$$

Then e) is true, and similarly f), because $x \neq 100$ (we know that x is less than 100).

---

4. *Question and Solution*

Estimate the product first, then do the multiplication.

a) *E:* 420 | 73 × 6 = 438

E: 450 | 146 × 3 = 438

E: 750 | 246 × 3 = 738

E: 1050 | 346 × 3 = 1038

b) *E*: 400 E: 450 E: 900 E: 750

	4	7	×	8
3	7	6		

1	4	7	×	3
4	4	1		

1	4	7	×	6
8	8	2		

2	4	7	×	3
7	4	1		

Notes

Your students should be familiar with the use of the word *product* meaning 'the result of a multiplication'.

Note that *E* is an estimation; there are no 'right' answers to this part of the question.

In the first part of a), an estimate is reached by calculating $70 \times 6\ (= 420)$, or the calculation could have been $75 \times 6\ (= 150 \times 3 = 450)$ or even $80 \times 5\ (= 400)$. Any of these answers are correct as estimations. You are just looking for a number that is reasonably close to the actual answer.

In the next calculation, *E* is found from $150 \times 3\ (= 450)$ and then $250 \times 3\ (= 750)$ and $350 \times 3\ (= 1050)$.

In b), the estimates can be calculated from

$50 \times 8\ (= 400)$ $150 \times 3\ (= 450)$ $150 \times 6\ (= 900)$ $250 \times 3\ (= 750)$

---

5. *Question and Solution*

Write the whole numbers up to 1000 which have 4 as the sum of their digits.

4, 13, 22, 31, 40, 103, 112, 121, 130, 202, 211, 220, 301, 310, 400

Notes

You need to consider what additions of four digits equals 4; for example,

1, 1 and 2	⇒	112, 121, 211
2, 2 and 0	⇒	22, 202, 220
1, 0 and 3	⇒	103, 130, 310, 301, 13, 31
4, 0 and 0	⇒	400, 40, 4

---

6. *Question and Solution*

Are the statements true or false? Write T for true and F for false in each box.

a) Every number which is a whole hundred is divisible by 2. T

b) There is an even number which has 5 as its units digit. F

c) Every number which is divisible by 5 is a whole ten. F

d) 217 is divisible by neither 5 nor 2. T

e) Every number which is a whole ten is divisible by 2 and by 5. T

Notes

a) The whole hundreds are 100, 200, 300, 400, 500, 600, 700, 800, 900 and all are divisible by 2 (in fact, any number ending in the digit zero is divisible by 2).

b) Any number ending in 5 is not divisible by 2 and so is ODD, not EVEN; hence the statement is false.

c) This is false as, for example, 25 is divisible by 5 but is not a whole ten.

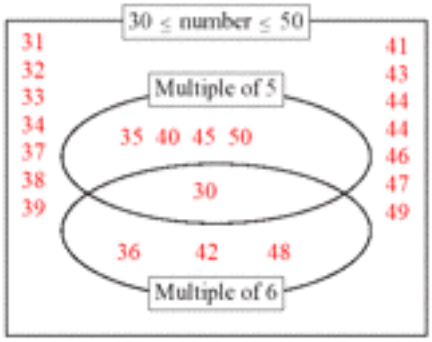
d) 217 is not divisible by 5 (as it does not end in 5 or 0) and not divisible by 2 (as it ends in an odd digit). Hence the statement is true (in fact, $217 = 7 \times 31$ so it is divisible by 7 and 31 as well as by 1 and 217).

e) Every number ending in 0 is divisible by 2 and 5 as $2 \times 5 = 10$.

---

7. *Question and Solution*

Write the whole numbers from 30 to 50 in the correct set.



Notes

Only the number 30 is divisible by both 5 and 6 and so is positioned in the intersection of the two sets.

Other multiples of 5, that is 35, 40, 45 and 50, are written in the 'Multiple of 5' set.

The multiples of 6, that is 36, 42 and 48, belong in the 'Multiple of 6' set.

The remaining numbers are written in the diagram, but outside the two sets.

---

8. *Question and Solution*

Write in the boxes the numbers described.

a) The smallest 4-digit: i) number [1000] ii) odd number [1001]

b) The greatest 4-digit: i) number [9999] ii) odd number [9999]

c) The greatest 4-digit number divisible by: i) 5 [9995] ii) 10 [9990]

d) The greatest 4-digit number divisible by 100 which has the same digit in its hundreds and thousands columns. [9900]

Notes

a), b), c) should be straightforward; for d), the possible answers, in numerical order, are

1100, 2200, 3300, 4400, 5500, 6600, 7700, 8800, 9900

and clearly the greatest is 9900.

---

9. *Question and Solution*

Estimate quickly, then calculate the sum.

a) 2653 + 1746

E: 2700 + 1700 = 4400

C:

	2	6	5	3
+	1	7	4	6
	4	3	9	9

b) 1256 + 7902

E: 1300 + 7900 = 9200

C:

	1	2	5	6
+	7	9	0	2
	9	1	5	8

c) 5343 + 2145

E: 5300 + 2100 = 7400

C:

	5	3	4	3
+	2	1	4	5
	7	4	8	8

Notes

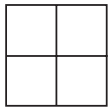
Again, it should be stressed that estimates can vary. For example, in a), we could use

$$3000 + 2000 \ (= 5000)$$

This is not a very accurate estimate but working to the nearest 1000, it is acceptable.

---

10. *Question and Solution*



a) How many rectangles are in this diagram? 9

b) How many rectangles would be in 874 such diagrams? 7866

c) What is the **area** of the diagram? $A =$ 4 square units

d) What is the **perimeter** of the diagram? $P =$ 8 units

Notes

a) You can see 4 small squares, , and one large one,  , (squares are also rectangles)

plus two rectangles with longer horizontal sides,  , and two with longer vertical  sides, .

This gives a total of 9 rectangles.

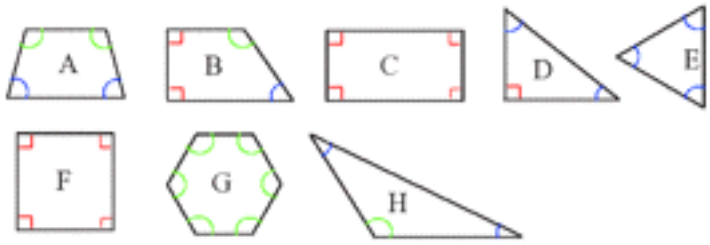
Parts b), c) and d) are straightforward.

---

11. *Question and Solution*

a) In each diagram, mark

- right angles in *red* like this, 
- angles **smaller** than a right angle in *blue* like this, 
- angles **larger** than a right angle in *green* like this. 



b) List the letters of the shapes for which each statement is true.

i) It is a square. F

ii) It is a rectangle. C, F

iii) It is a quadraliteral. A, B, C, F

iv) It is a triangle. D, E, H

v) It has at least one right angle. B, C, D, F

vi) Every angle is a right angle. C, F

vii) It has at least one angle smaller than a right angle. A, B, D, E, H

viii) All its angles are smaller than a right angle. E

ix) It has at least one angle larger than a right angle. A, B, G, H

x) All its angles are larger than a right angle. G

Notes

Part a) is straightforward but in part b), note that both C and F are rectangles, whilst A, B, C and F are all quadrilaterals (that is, they are closed shapes formed by 4 straight lines).

---

12. *Question and Solution*

Complete these non-convex shapes so that they become **convex** shapes.

Notes

The question would have been better worded as

"Change these shapes into convex shapes."

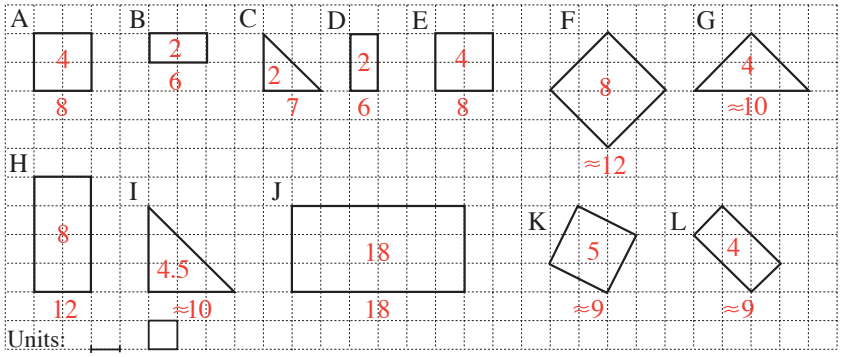
There are many possible correct answers!

---

13. *Question and Solution*

List the **similar** shapes.

Write the **area** inside each shape and the length of the **perimeter** below.



Similar shapes: A E F K; B D H J L; C G I

Notes

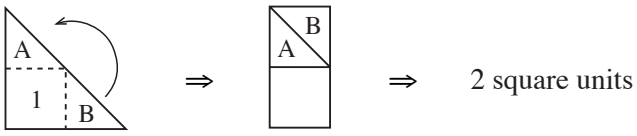
There are 3 distinct sets of similar shapes:

A, E, F and K - these are all squares

B, D, H, J and L - these are all rectangles with side lengths in the ratio 2 to 1, i.e. 2 : 1
(note that in J the ratio is 6 : 3 - this is the same as 2 : 1)

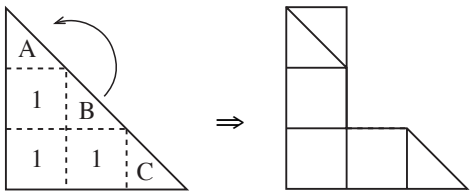
C, G and I - these are all triangles with a right angle and two equal sides enclosing the right angle.

The areas are straightforward, but note that for C, we can work out the area by moving part of the shape:



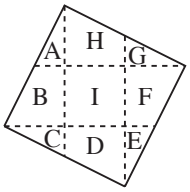
$\Rightarrow$ $\Rightarrow$ 2 square units

For I, we can redraw in this way:



$\Rightarrow$ $\Rightarrow$ 4 and a half $\left(4\frac{1}{2}\right)$ square units

K is more complicated:



$\Rightarrow$

$$\left.\begin{array}{l} A + B = 1 \\ C + D = 1 \\ E + F = 1 \\ G + H = 1 \\ \text{middle} = 1 \end{array}\right\}$$ 5 square units

For the perimeters, some need to be estimated, that is, C, J, K and L.

---

14. *Question and Solution*

Complete the fractions.

a) $\frac{1}{2} = \frac{\boxed{2}}{4} = \frac{4}{\boxed{8}} = \frac{\boxed{3}}{6} = \frac{\boxed{5}}{10} = \frac{10}{\boxed{20}} = \frac{\boxed{50}}{100} = \frac{\boxed{100}}{\boxed{200}}$

b) $\frac{1}{4} = \frac{\boxed{4}}{16} = \frac{2}{\boxed{8}} = \frac{\boxed{5}}{20} = \frac{8}{\boxed{32}} = \frac{25}{\boxed{100}} = \frac{\boxed{50}}{\boxed{200}} = \frac{\boxed{3}}{\boxed{12}}$

c) $\frac{1}{3} = \frac{2}{\boxed{6}} = \frac{\boxed{4}}{12} = \frac{3}{\boxed{9}} = \frac{\boxed{5}}{15} = \frac{\boxed{8}}{24} = \frac{\boxed{10}}{30} = \frac{100}{\boxed{300}} = \frac{\boxed{6}}{\boxed{18}}$

Notes

Note that in a), to get from the first fraction, $\frac{1}{2}$, to any of the other 'equivalent' fractions, you multiply 'top' (numerator) and 'bottom' (denominator) by the same number. For example,

$$\frac{1}{2} \overset{\times 3}{=} \frac{3}{6} \qquad \text{or} \qquad \frac{1}{2} \overset{\times 100}{=} \frac{100}{200}$$

At this level, we say that all the fractions are equal but the technical term is 'equivalent fractions', and $\frac{1}{2}$ is the *simplest form* of all the fractions in a).

---

15. *Question and Solution*

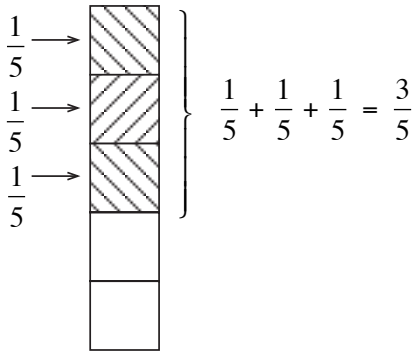
a) $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \boxed{\frac{3}{5}}$ b) $\frac{3}{8} + \frac{2}{8} = \boxed{\frac{5}{8}}$ c) $\frac{7}{12} - \frac{2}{12} = \boxed{\frac{5}{12}}$

d) $\frac{11}{20} - \frac{9}{20} = \boxed{\frac{2}{20}}$ e) $\frac{7}{10} + \frac{3}{5} = \boxed{\frac{13}{10}}$ f) $\frac{3}{4} - \frac{3}{8} = \boxed{\frac{3}{8}}$

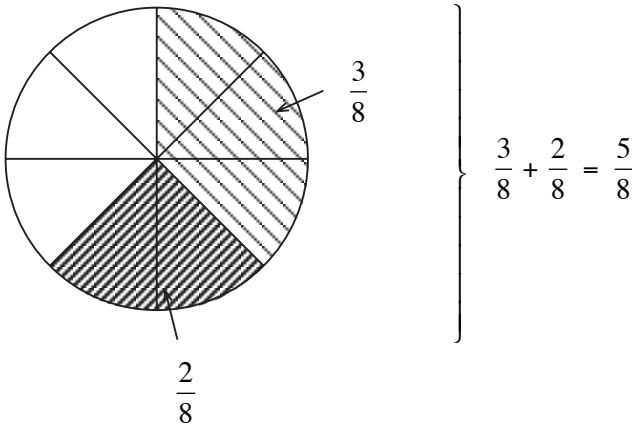
Notes

This exercise introduces the formal addition of fractions. You can easily illustrate the answers with diagrams. For example,

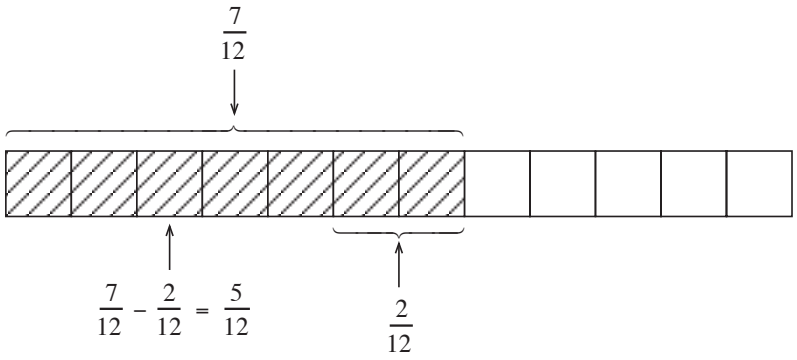
a)



b)



Subtraction follows in a similar way:



Parts e) and f) require the use of equivalent fractions to ensure that the denominators are the same, that is,

e) $\frac{7}{10} + \frac{3}{5} = \frac{7}{10} + \frac{6}{10} = \frac{13}{10}$ (as $\frac{3}{5} = \frac{6}{10}$)

f) $\frac{3}{4} - \frac{3}{8} = \frac{6}{8} - \frac{3}{8} = \frac{5}{8}$ (as $\frac{3}{4} = \frac{6}{8}$)

---

16. *Question and Solution*

Compare the pairs of numbers and fill in the missing signs. (<, >, =)

Use the diagrams to help you.

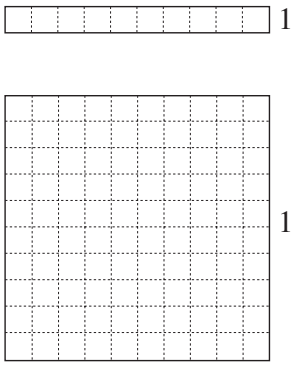
a) $\frac{2}{10} < \frac{7}{10}$ $\frac{8}{10} < 0.9$ $0.6 > 0.3$

b) $\frac{15}{100} < \frac{72}{100}$ $\frac{43}{100} < 0.70$ $0.52 > 0.49$

c) $0.04 < 0.1$ $\frac{2}{10} > \frac{18}{100}$ $0.27 < 0.3$

d) $\frac{1}{5} = 0.2$ $\frac{2}{5} < 0.3$ $\frac{3}{10} < 0.6$

e) $\frac{1}{5} > \frac{17}{100}$ $\frac{3}{10} < 0.51$ $\frac{78}{100} > 0.53$



Notes

This exercise involves both fractions and decimals. Any decimal can be written as a fraction.

For example,

$$0.9 = \frac{9}{10}$$

$$0.04 = \frac{4}{100}$$

and any fraction can be written as a decimal by forming an equivalent fraction with one of 10, 100, 1000, etc. as the denominator.

For example,

$$\frac{2}{5} = \frac{4}{10} = 0.4$$

$$\frac{1}{4} = \frac{5}{100} = 0.05$$

Hence in a) $\frac{8}{10} < 0.9 \left(= \frac{9}{10}\right)$ and in b) $\frac{43}{100} < 0.70 \left(= \frac{70}{100}\right)$,

whilst in d) $\frac{1}{5} = 0.2 \left(\text{as } \frac{1}{5} = \frac{2}{10} = 0.2\right)$

---

17. *Question and Solution*

Which quantity is greater? Fill in the missing signs.

a) $\frac{3}{10}$ m $\boxed{<}$ 54 cm b) 0.9 kg $\boxed{>}$ 90 g c) $\frac{1}{6}$ hour $\boxed{<}$ 30 min

d) £150 20 p $\boxed{=}$ £150.2 e) $5\frac{7}{100}$ litres $\boxed{<}$ 5 litres 700 ml

f) $4\frac{1}{2}$ weeks $\boxed{<}$ 29 days g) 84.3 cm $\boxed{=}$ 843 mm $\boxed{<}$ 8.43 m

Notes

Now we have not only fractions and decimals but also units to consider.

a) $\frac{3}{10}$ m $= \frac{30}{100}$ m $= 30$ cm < 54 cm

b) 0.9 kg $= 0.9 \times 1000$ g $= 900$ g > 90 g

c) $\frac{1}{6}$ hours $= 60$ min $\div 6 = 10$ min < 30 min

d) £150 20 p can be written as £150.20 which is £150.2 as a decimal number (although we do not normally write it like this).

e) $5\frac{7}{1000}$ litres $= 5$ litres 7 ml $<$ 5 litres 700 ml

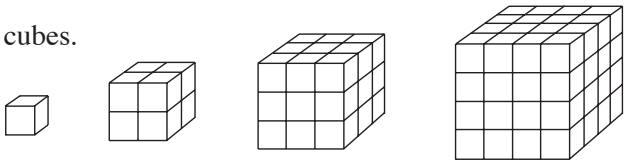
f) $4\frac{1}{2}$ weeks $= \left(4 \times 7 + \frac{1}{2}\right)$ days $= 28\frac{1}{2}$ days < 29 days

g) 84.3 cm $= 843$ mm $= 0.843$ m < 8.43 m

---

18. *Question and Solution*

Imagine these cubes built from unit cubes.
Fill in the missing numbers.



Length of 1 edge ⟼	1	2	3	4	5	6
Area of cube □	6	24	54	96	150	216
Volume of cube	1	8	27	64	125	216

Notes

This is a straightforward question but note the pattern emerging.

For the area, we have

$$6 \times 1,\quad 6 \times 4,\quad 6 \times 9,\quad 6 \times 16,\quad 6 \times 25,\quad 6 \times 36,\quad \ldots,$$

The numbers 1, 4, 9, 16, . . ., are called 'square numbers' as

$$1 = 1 \times 1,\quad 4 = 2 \times 2,\quad 9 = 3 \times 3,\quad \text{etc.}$$

The volume pattern is

$$1,\quad 8,\quad 27,\quad 64,\quad 125,\quad 216,\quad \ldots,$$

and these are 'cubic numbers', that is,

$$1 = 1 \times 1 \times 1,\quad 8 = 2 \times 2 \times 2,\quad 27 = 3 \times 3 \times 3,\ \text{etc.}$$

---

19. *Question and Solution*

Follow the example. Complete the sentences. Use the number line to help you.

b) 3°C is [less] than 8°C by 5°C. $3 - 8 =$ [−5], [−5] $+ 8 = 3$

c) 8°C is greater than 0°C by [8°C]. $8 - 0 =$ [8], [8] $+ 0 = 8$

d) 3°C is greater than −2°C by [5°C]. $3 - (-2) =$ [5], [5] $+ (-2) = 3$

e) −2°C is less than 3°C by [5°C]. $-2 - 3 =$ [−5], [−5] $+ 3 = -2$

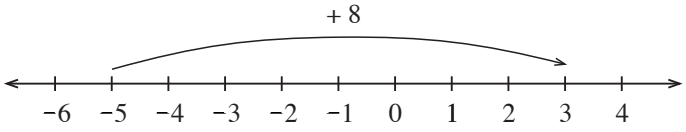
f) −2°C is [greater] than −5°C by 3°C. $-2 - (-5) =$ [3], [3] $+ (-5) = -2$

Notes

Dealing with addition/subtraction of negative numbers can often be confusing so it helps if you illustrate what you are doing on a number line.

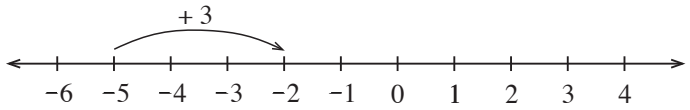
For example,

b)



$$-5 + 8 = 3$$

d)



−2 is 3 greater than −5

(In context, we are showing that a temperature of −5°C is 3°C colder than −2°C.)

---

20. *Question and Solution*

In an opaque bag there are 10 *black* and 30 *white* marbles.

What is the smallest number of marbles you must take out of the bag (with your eyes closed) to be **certain** of getting 2 marbles which are the same colour?

3

Notes

This is almost a 'trick question': many people will give the answer '11'.

It is clearly 3, as the worst outcome is to obtain

white	*black*

or

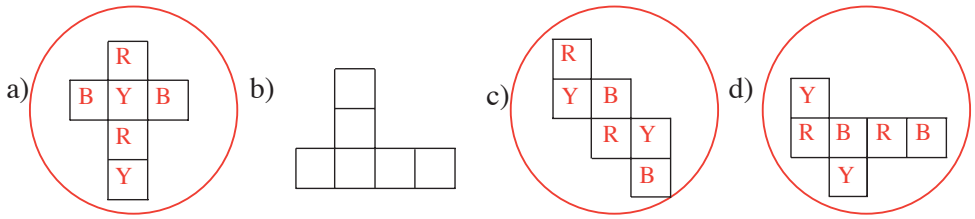
black	*white*

for the first two marbles. The third one taken must be either *black* or *white*, giving 2 marbles that are the same colour.

---

21. *Question and Solution*

Circle the nets which can make a cube. Colour their opposite faces in the same colour.



Notes

It is always best to cut out models of the nets and try them out. Some students will be able to visualise without the physical model, but going through this process is valuable for many.

---

22. *Question and Solution*

What is the smallest natural multiple of 2, 3, 4, 5 and 8?

120

Notes

Here you are looking for the smallest whole number that can be divided exactly by 2, 3, 4, 5 and 8 (no remainder).

Both 2 and 4 are factors of 8, so really we are finding the smallest multiple of 3, 5 and 8.

These numbers have no common factor, so the number required must be

$$3 \times 5 \times 8 = 120$$

---

23. *Question and Solution*

In an opaque bag, there are 5 *black*, 10 *red* and 5 *white* marbles.

What is the smallest number of marbles you must take out of the bag (with your eyes closed) to be **certain** of getting:

a) 3 marbles which are the same colour

b) a *red* marble?

a) 7 b) 11

Notes

a) The worst situation is to obtain 2B, 2R and 2W for the first 6 marbles taken.

The next one has to give you 3 of the same colour, whatever colour it is.

b) Think very carefully about this question. The worst scenario is to first take all the B and W marbles, that is 10 marbles. The next one *must* be R!

---

24. *Question and Solution*

List in your exercise book all the numbers between 999 and 10 000 which have 4 as the sum of their digits. How many did you find?

20

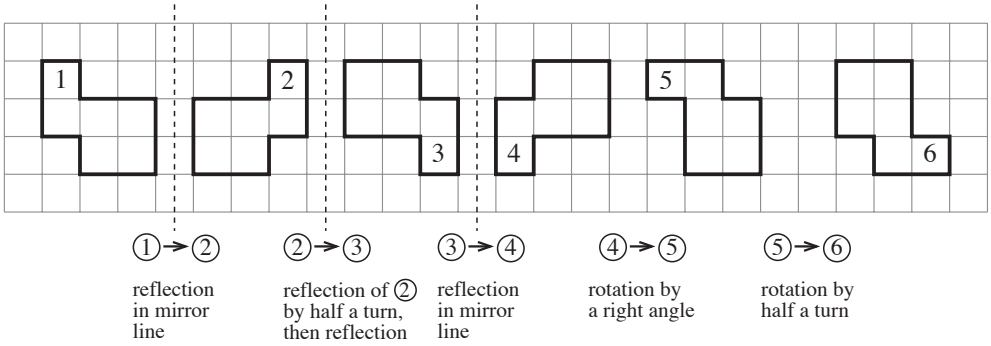
Notes

Be systematic in finding all the possible answers. The full list is shown below.

'1, 1, 1, and 1' ⇒	1 1 1 1	(1)
'1, 1, 2 and 0' ⇒	2 1 1 0, 2 1 0 1, 2 0 1 1, 1 2 1 0, 1 2 0 1, 1 1 2 0, 1 1 0 2 1 0 1 2, 1 0 2 1	(9)
'2, 2, 0 and 0' ⇒	2 2 0 0, 2 0 2 0, 2 0 0 2	(3)
'1, 3, 0 and 0' ⇒	3 1 0 0, 3 0 1 0, 3 0 0 1 1 3 0 0, 1 0 3 0, 1 0 0 3	(6)
'4, 0, 0 and 0' ⇒	4 0 0 0	(1)
		20

25. *Question and Solution*

These shapes are **congruent**. What has been done to *Shape 1* to make *Shape 2*, *Shape 2* to make *Shape 3*, and so on? Write it in your exercise book.



Notes

Note that

half a turn = 2 right angles

quarter turn = 1 right angle

It might be helpful to use tracing paper for the rotations.

---

26. *Question and Solution*

Among 67 scientists at a conference,

47 speak French,

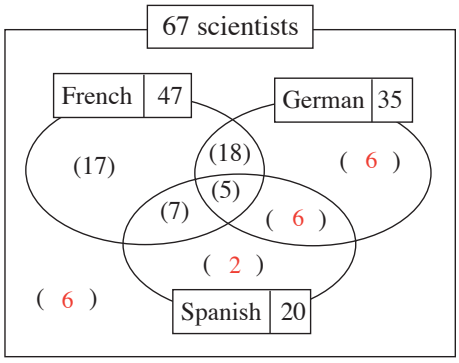
35 speak German,

20 speak Spanish,

12 speak French and Spanish,

11 speak German and Spanish,

5 speak all three languages.



a) Complete the *Venn* diagram.

b) How many scientists speak:

i) only French 17 ii) only German 6 iii) only Spanish? 2

c) How many scientists speak Spanish and German but not French? 6

d) How many scientists speak neither Spanish nor German nor French? 6

Notes

To complete the Venn diagram, note that

11 speak G and S $\Rightarrow$ 6 speak G and S but not F

so 6 can be inserted.

Looking at G, the total is 35, so the missing number is

$$35 - (18 + 5 + 6) = 35 - 29 = 6$$

Similarly, 20 is the total for S, and the missing number is

$$20 - (7 + 5 + 6) = 20 - 18 = 2$$

Finally, there are 67 scientists in total. Adding up all the numbers inserted gives

$$17 + 18 + 5 + 7 + 6 + 6 + 2 = 61$$

So there are $67 - 61 = 6$ in the outside region of the Venn diagram.

---

27. *Question and Solution*

Circle the natural numbers up to 100 which have only two factors.
(e.g. the only factors of **7** are 7 and 1)

1	(2)	(3)	4	(5)	6	(7)	8	(9)	10	(11)	12	(13)	14	15	16	(17)	18	(19)	20
21	22	(23)	24	25	26	27	28	(29)	30	(31)	32	33	34	35	36	(37)	38	(39)	40
(41)	42	43	44	45	46	(47)	48	49	50	51	52	(53)	54	55	56	57	58	(59)	60
(61)	62	63	64	65	66	(67)	68	69	70	(71)	72	73	74	75	76	77	78	(79)	80
81	82	(83)	84	85	86	87	88	(89)	90	91	92	93	94	95	96	(97)	98	99	100

List these numbers in increasing order.

2, 3, 5, 7, 9, 11, 13, 17, 19, 23, 29, 31, 37, 39, 41, 47, 53,
59, 61, 67, 71, 79, 83, 89, 97

Notes

Numbers with only 2 factors are called PRIME numbers and are very important in the development of mathematics. Although it appears that they occur less frequently as the numbers get larger, this is not the case, and they keep on occurring as there is an *infinite* number of them.

Many recent applications of prime numbers have occurred, including a method of coding and decoding electronic messages using two very large prime numbers multiplied together.

---

28. *Question and Solution*

The perimeter of a triangle is 10 cm and the length of each side is a whole cm.

Are these statements true or false? Write a ✓ if true and a **X** if false.

a) The triangle has only one side which is 1 cm long. X

b) The triangle could have only one side which is 2 cm long. ✓

c) The triangle has only one side which is 3 cm long. X

d) The triangle has only one side which is 5 cm long. X

Notes

a) If a triangle has only one side of length 1 cm, then the possible lengths could be

1 cm, 8 cm, 1 cm
1 cm, 7 cm, 2 cm } it is clearly not possible to draw these triangles!
1 cm, 6 cm, 3 cm

1 cm, 5 cm, 4 cm - this is a straight line.

b) The triangle could have sides of

2 cm, 4 cm, 4 cm

c) The possible lengths are

3 cm, 6 cm, 1 cm ← no triangle
3 cm, 5 cm, 2 cm ← straight line
3 cm, 4 cm, 3 cm ← this has 2 sides of length 3 cm, so is not allowed

d) Here you can have

5 cm, 4 cm, 1 cm
5 cm, 3 cm, 2 cm } both straight lines!

Mathematics Enhancement Programme

TEACHING SUPPORT: Book 4

INTRODUCTION TO LESSON PLANS

Detailed Lesson Plans* have been developed to be used as guidance for *MEP Primary* teaching. They are based on

4 × 45 minute lessons per week for 35 weeks

The online Revision Questions are for you to use for learners' practice and revision, perhaps for the fifth lesson of the week or for homework.

The Lesson Plans have been adapted from the Eastern European approach to mathematics teaching. The first few weeks of the course provide introductory activities to help you assess the capabilities of your class and to bring them together at a suitable starting point for the systematic treatment.

The course provides a very strong mathematical foundation, dealing with topics in depth and in a logical sequence, and using appropriate notation. It provides the necessary background in mathematical logic, aiming to give enhanced attainment in national tests and a solid framework for higher level work.

The Lesson Plans make reference to:

- Practice Books (e.g. *Practice Book 3*)

and make use of a range of resources:

- number cards (0 to 9)
- symbol cards (=, <, >, ≤, ≥, +, −, ×, ÷, ≠)
- shape cards (◯, △, □, ⬠, ⬡ / large and small / black and white
- number lines (−9 to 0, 0 to 9, 10 to 20, 0 to 100)
- number strips or rods or plastic cubes stuck together
- dominoes
- toy money
- coloured counters
- coloured sticks/straws

Reference is made to a number of A2 posters *. We would also like you to encourage learners to collect their own materials for use in maths lessons (e.g. pebbles, shells, buttons, sweets, trinkets) to make maths more relevant to them individually.

* online at http://www.cimt.org.uk/projects/mepres/primary/index.htm

Year	Week beginning	*Order of Topics*
	Sept	Familiarisation with Y4 resources. (IPMA **Test 3** for Ps who did not do it at the end of Y3)
		1. Revision: numbers up to 1000: writing and ordering, comparison, rounding, sequences
		2. Revision: Operations with numbers up to 1000. Addition and subtraction
		3. Revision: Operations with numbers up to 1000. Multiplication and division tables
	Oct	4. Revision: Written calculations. Division with remainders. Divisibility
		5. Revision: length, capacity, mass, time. Numbers up to 2000
		6. Revision and practice: Numbers and calculations up to 2000
		Half Term
	Nov	7. Extending numbers to 10 000: counting, reading, writing, ordering, rounding, number line
		8. Addition and subtraction up to 10 000. Written calculations
		9. Multiplication and division up to 10 000. Written calculations. ($\times$ and $\div$ by 1-digit)
		10. Geometry: grouping 1-D, 2-D, 3-D objects. Angles. Parallel and perpendicular lines
		11. Shapes: Properties, angles; parallel and perpendicular lines. Convex and concave.
	Dec	12. Shapes and solids: similarity and congruence, reflection, symmetry
		13. Revision and practice: multiplication and division by 1-digit numbers
		Christmas
	Jan	14. Revision and practice: 4 operations, geometry
		15. Problems in context
		16. Fractions, including tenths. Equivalent fractions. Position on number line
		17. Fractions: addition and subtraction (equal demonimators)
	Feb	18. Review and practice: Natural numbers and fractions. Equations
		19. Fractions and decimals. Decimal notation
		Half Term
		20 Addition and subtraction of decimals (1 decimal place)
	Mar	21. Fractions and decimals in context. Problems in length, capacity, mass, time, money
		22. Perimeter , area, volume (with natural numbers, fractions and decimals)
		23. Practice: addition, subtraction, multiplication, division
		24. Natural numbers up to 10 000. Rounding
		25. Problems in context. Measures. Fractional parts
	Apr	**Easter**
		26. Positive and negative numbers: thermometers, number line. Comparisons
		27. Positive and negative numbers. Money: cash and debt. Addition and subtraction
	May	28. Revision and practice: numbers, word problems, factors, multiples
		29. Revision and practice: geometry, geometric games and puzzles
		30. Collecting and displaying data. Tally charts and grouping
		31. Data, diagrams, tables, functions, single line graphs
		Half Term
	Jun	32. Probability: fair and unfair games, experiments
		33. Revision and practice
		34. Revision and practice
		35. Puzzles and challenges
	Jul	**IPMA Test 4**

Year 4 Contents Page
Lessons 1 to 30

R: Revision- Not included **C: Core** *E: Extension*

Year 4 Contents Page
Lessons 31 to 50

R: Revision- Not included **C: Core** *E: Extension*

Year 4 Contents Page
Lessons 51 to 80

R: Revision- Not included **C: Core** *E: Extension*

Year 4 Contents Page
Lessons 81 to 110

R: Revision- Not included **C: Core** *E: Extension*

Year 4 Contents Page
Lessons 111 to 140

R: Revision- Not included **C: Core** *E: Extension*

Year 4 Contents Page
Lessons 141 to 175

R: Revision- Not included **C: Core** *E: Extension*