Mathematics Enhancement Programme (MEP)

PRIMARY Book 3

Support for Parents, Carers and Teachers

Yearly Objectives with Examples

Key Facts and Glossary

Problem Solving Examples

Overview of Contents

Detailed Contents

Mathematics Enhancement Programme

TEACHING SUPPORT

GENERAL OVERVIEW

This initiative, the *Mathematics Enhancement Programme, Primary Project*, has been developed by the Centre for Innovation in Mathematics Teaching (CIMT) at Plymouth University to enhance the mathematical progress of learners in primary schools. It is based on evidence of good practice from mathematically high performing countries.

MEP aims to support these KEY STRATEGIES for effective mathematics teaching and learning:

1. **Lesson** well prepared (teacher knows the lesson plan well and is aware of problems/difficulties that are likely to occur), resources are at hand, board prepared in advance, learners have their own resources on desk.

2. **Seating**: learners have eye contact with the teacher and can get to the board quickly and easily.

3. **Whole-class interactive teaching** predominates, with planned intervals of individual and paired work. Learners on task and given the chance to demonstrate, answer, explain to the class.

4. **Friendly, non-confrontational atmosphere** where learners learn from and support others and have fun! Mistakes used as teaching points. Encouragement given to those who have difficulty and praise given when deserved. Learners are encouraged to appreciate the good work of others.

5. **Spiral curriculum** with continual revision; learning by heart encouraged, with progression in small, logical steps.

6. **Visualisation** and **manipulatives** are used in the early years and less able learners. Relating contexts to learners' experiences, demonstrate on number lines, modelling to help understanding.

7. **Exercises** reviewed interactively with the whole class at the time. Learners give the solutions, not the teacher, and rest of the class agrees/disagrees or suggests alternative solutions. Learners are expected to correct their own work. Teacher gives hints only if the whole class is stuck.

8. **Challenges** or **extension work** set for able learners, or they help less able neighbours.

9. Introductory and Reinforcement tasks to help ensure that no children are left behind.

10. **Correct notation, layout** and **language** used at all times. Teacher acts as a model for learners to follow (on board and orally), repeating/showing a learner's explanation more clearly and succinctly where necessary.

11. **Good pace** and **varied activities** related to the concept being taught. Time limits set for individual/paired work. Time allowed for learners to explain and for whole class discussion.

12. Assessment, formative and summative, based on the key objectives in the National Curriculum

The course is fully resourced with:

- Learner Practice Books, with detailed lesson plans for teachers
- Number cards, shape cards and number lines
- Interactive resources for reinforcement, revision and extension

Resources are freely available at: http://www.cimt.org.uk/projects/mepres/primary/

Mathematics Enhancement Programme

TEACHING SUPPORT: Book 3

LEARNING OBJECTIVES

At the end of Year 3 we expect learners to

- *use numbers up to thousands with confidence*
- *have instant recall of multiplication facts up to* 10×10
- *understand and calculate half, quarters, thirds, eights, etc. of simple quantities, including time*
- *understand the concept of negative numbers in context*
- *round numbers to the nearest 10 or 100*
- *use units of money (£ and p), length (km, m, cm), weight (kg, g), capacity (l, cl, ml) and time (seconds, minutes, hours, days, weeks, months, years)*
- *know the names and properties of familiar shapes (points, lines, triangles, quadrilaterals, rectangles, squares, spheres, cubes and cuboids)*
- *calculate the perimeter and area of simple shapes (rectangles)*
- *understand and use reflections and enlargements*
- *be familiar with similar and congruent shapes*
- *use bar charts and pictograms to illustrate data*
- *recognise the need for brackets in complex calculations.*

Mathematics Enhancement Programme

TEACHING SUPPORT: Book 3

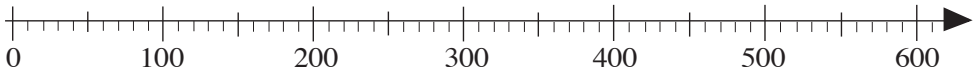
LEARNING OBJECTIVES with ILLUSTRATIVE EXAMPLES (and Answers appended)

At the end of Book 3 we expect learners to be confident with the following concepts.

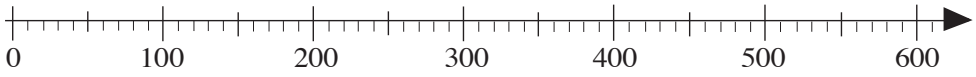
Use numbers up to thousands with confidence.

Example 1 Which whole numbers make each statement true? Mark them on the number line. Write down the highest and lowest possible numbers.

a) 380 < ☐ < 450 ☐ : ..... to .....



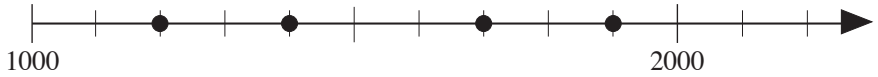
b) 280 ≤ ☐ ≤ 380 ☐ : ..... to .....



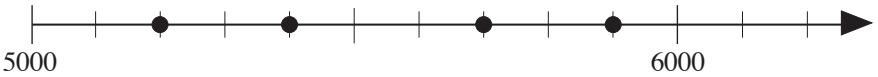
Example 2 Write the numbers below the dots.

a)

b)



c)



Example 3 List all the 3-digit numbers in which:

a) the sum of the 3 digits is 5, ...

b) the product of the 3 digits is 4, ...

c) the sum of the 3 digits is 4. . . . . .

Have instant recall of multiplication facts up to 10×10***.***

Example 4 Fill in the missing products. Note how they change.

a) $60 \times 3 =$ ☐ $60 \times 6 =$ ☐ $60 \times 9 =$ ☐ $60 \times 12 =$ ☐

b) $40 \times 5 =$ ☐ $40 \times 10 =$ ☐ $40 \times 15 =$ ☐ $40 \times 25 =$ ☐

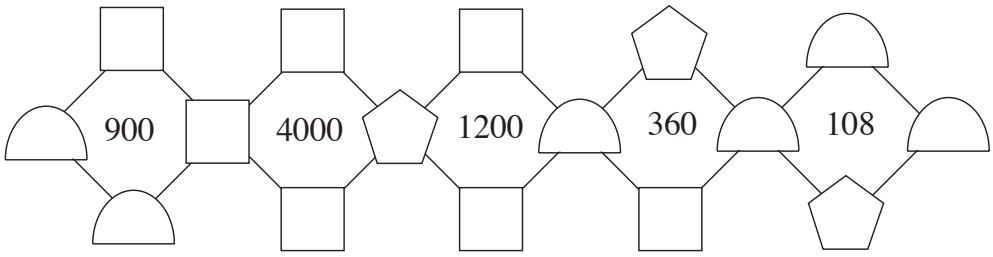
c) $4 \times 2 =$ ☐ $40 \times 2 =$ ☐ $400 \times 2 =$ ☐ $40 \times 20 =$ ☐

d) $3 \times 5 =$ ☐ $30 \times 5 =$ ☐ $300 \times 5 =$ ☐ $30 \times 50 =$ ☐

e) $4 \times 24 =$ ☐ $8 \times 12 =$ ☐ $16 \times 6 =$ ☐ $2 \times 48 =$ ☐

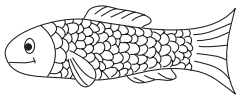
$4 \times 240 =$ ☐ $8 \times 120 =$ ☐ $16 \times 60 =$ ☐ $2 \times 480 =$ ☐

Example 5 The middle number is the product of the 4 numbers around it. Fill in the missing numbers.



Understand and calculate half, quarters, thirds, eights, etc. of simple quantities, including time.

Example 6 *Sparrow* and *Trout* were arguing over the times in a day. Who is correct? Tick the correct answer and cross out the wrong one.



12 hours	half a day	30 hours
14 hours	2 quarters of a day	12 hours
4 hours	1 sixth of a day	4 hours
45 minutes	2 half hours	60 minutes
15 minutes	a quarter of an hour	20 minutes
40 minutes	2 thirds of an hour	45 minutes
2 hours	1 eighth of a day	3 hours
9 hours	2 sixths of a day	8 hours
18 minutes	3 tenths of an hour	20 minutes

Example 7 Fill in the missing numbers. ('min' means 'minutes' and 'hrs' means 'hours')

a) half an hour = ☐ min

3 quarters of an hour = ☐ min

3 fifths of an hour = ☐ min

2 thirds of an hour = ☐ min

5 sixths of an hour = ☐ min

3 tenths of an hour = ☐ min

2 and a half hours = ☐ min

b) half a day = ☐ hrs

2 thirds of a day = ☐ hrs

3 quarters of a day = ☐ hrs

5 eighths of a day = ☐ hrs

1 twelfth of a day = ☐ hrs

1 and a half days = ☐ hrs

5 half days = ☐ hrs

Understand the concept of negative numbers in context.

Example 8 Wendy went to Austria for a winter holiday. One day, she decided to note down the outside temperature every hour. She made this table to show her data.

Time (hours)	7	8	9	10	11	12	13	14	15	16	17	18	19
Temperature (°C)	– 9	– 10	– 6	– 2	0	3	6	8	9	7	4	– 1	– 3

a) When was it: i) coldest . . . . . ii) warmest? . . . . . .

b) Write the temperatures in increasing order.

. . . . . .

Example 9 Are the inequalities correct? Mark with a ✔ or a ✕. Correct the mistakes.

a) $-8 < -2$ b) $-\square 20 > -\square 10$ c) $-5 < 5$ d) $-6 > -7$

e) $-10 < -9$ f) $-\square 15 > -\square 20$ g) $0 < -\square 1$ h) $-50 < 2$

Round numbers to the nearest 10 or 100.

Example 10 Two different numbers round to 300 as the nearest hundred. Is it possible that:

a) both numbers are less than 300 . . . . . .

b) the smaller number is 100 less than the other number . . . . . .

c) one number has 5 and the other has 0 as the tens digits . . . . . .

d) both numbers are whole hundreds? . . . . . .

Example 11 a) Complete the table.

Number	Rounded to nearest 10	Rounded to nearest 100
943		
304		
184		
765		
125		
550		
247		
805		

b) List all possible whole numbers which have:

- 5 as the tens digit when rounded to the nearest ten,

and also

- 5 as the hundreds digit when rounded to the nearest hundred.

Use units of money (£ and p), length (km, m, cm), weight (kg, g), capacity (l, cl, ml) and time (seconds, minutes, hours, days, weeks, months, years).

Example 12 Fill in the missing numbers.

a) 1 m 30 cm = ☐ cm

b) 1 m 50 cm = ☐ cm

c) 1 m 100 cm = ☐ m = ☐ cm

d) 1 m 26 cm = ☐ cm

e) 1 m 80 cm = ☐ cm

f) 1 m 7 cm = ☐ cm

Example 13 Fill in the missing numbers and units.

a) 1 kg = 1000 ☐

b) half a kg = ☐ g

c) . . . . . kg = 1500 g

d) . . . . . kg = 250 g

e) 1 tonne = 1000 ☐

f) half a tonne = ☐ kg

Example 14 Fill in the missing numbers and units.

a) 2 litres = 200 ☐

b) 5 litres = ☐ cl

c) 9 litres = ☐ cl

d) 3 litres 50 cl = 350 ☐

e) 2 and a half litres = ☐ cl

f) 40 cl = ☐ ml

Example 15 Complete the table.

ml	1200	2000	800				1850
cl	120					190	
10 cl	12				15		
litres	1 and 2 tenths			1 and 23 hundredths			

Example 16 Is it possible to answer the question with the data given? If it is, solve it.

a) 10 kg of bananas costs £9.40. What is the price of 1 kg of bananas?

. . . . . .

b) Steve bought 10 different bars of chocolate and paid £12.00 altogether.
What was the price of 1 bar of chocolate?

. . . . . .

c) Karen is 9 years old. She weighs 27 kg.
What did she weigh when she was 1 year old?

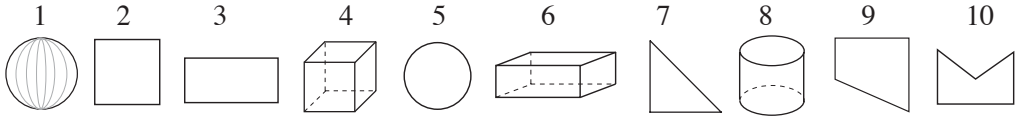
. . . . . .

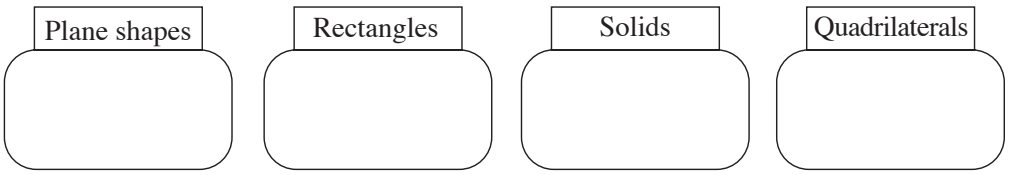
d) 3 men worked steadily and painted a 540 m fence in 9 days.
How many days would it have taken 1 man to paint the same fence?

. . . . . .

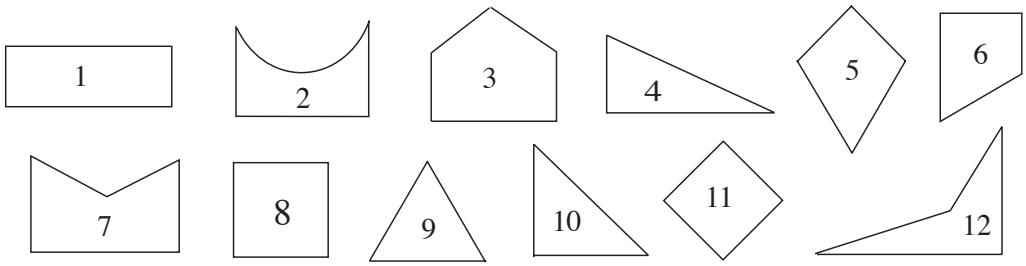
Know the names and properties of familiar shapes (points, lines, triangles, quadrilaterals, rectangles, squares, spheres, cubes and cuboids).

Example 17 Which shape belongs in which box? Write the numbers in the correct boxes.





Example 18 These **plane** shapes were cut out from coloured paper.



List the numbers of the shapes which are:

a) quadrilaterals: . . . . . .

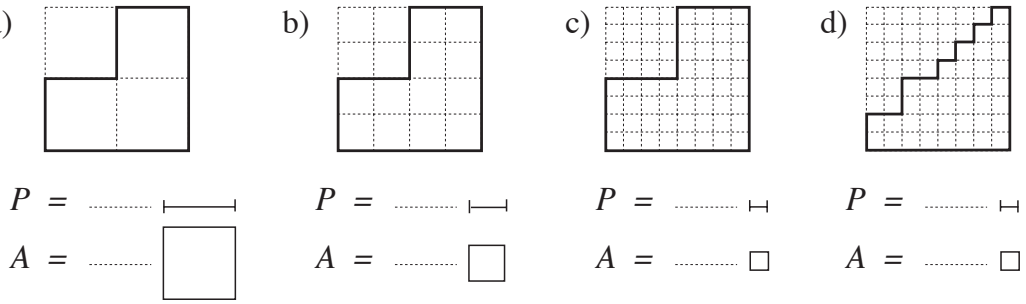
b) rectangles: . . . . . .

c) squares: . . . . . .

Calculate the perimeter and area of simple shapes (rectangles).

Example 19 Count how many of the given units are in the perimeter and area of each shape.

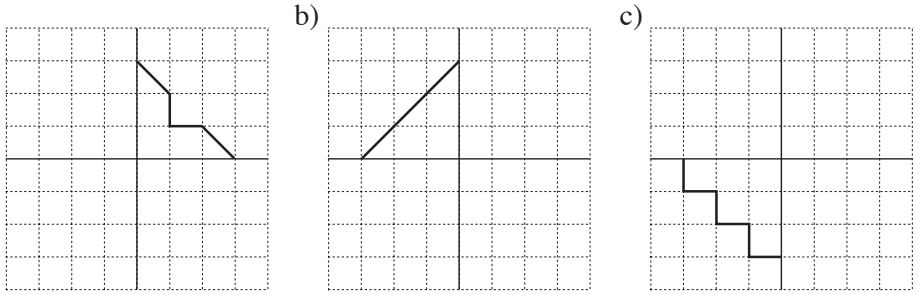
a) b) c) d)



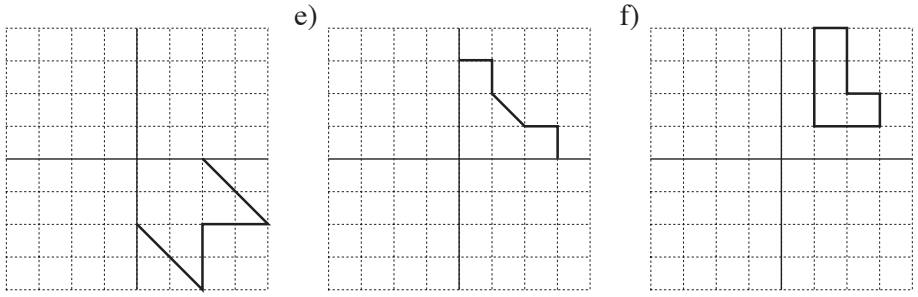
Understand and use reflections and enlargements.

Example 20 Reflect the shape in one **axis** first. Then reflect the shape and its mirror image in the other **axis**. Draw the mirror lines of the whole shape.

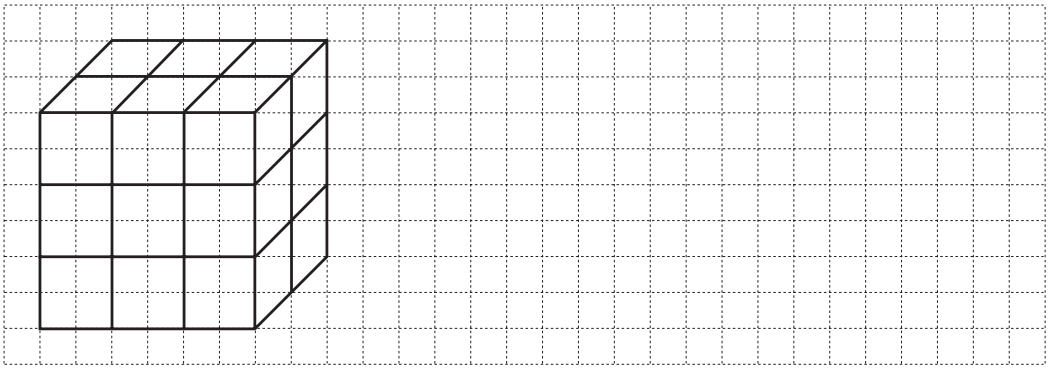
a) b) c)



d) e) f)



Example 21 a) Reduce this cuboid to: i) half its size ii) 1 third of its size.

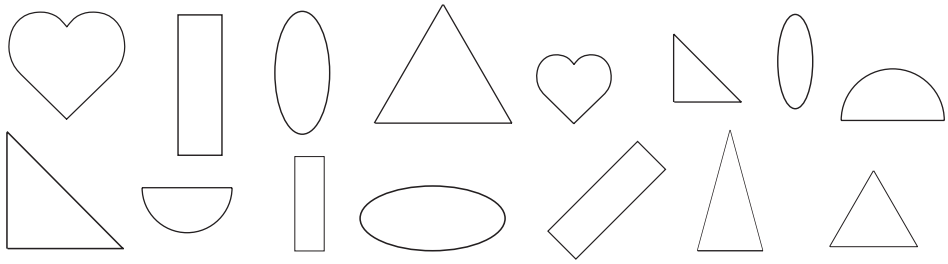


b) Enlarge this cuboid to: i) twice its size ii) 3 times its size.

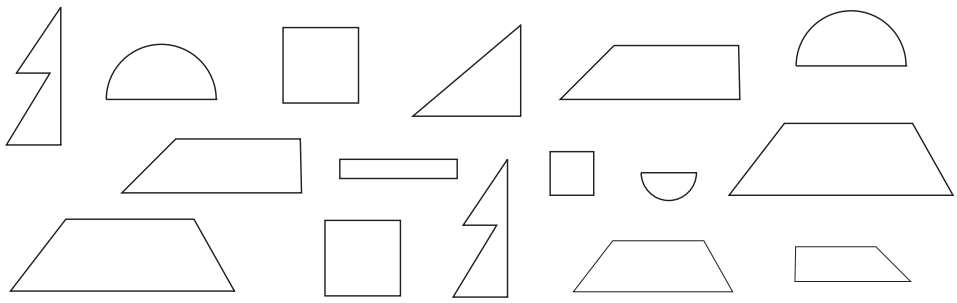
c) What is the volume of each of the 6 cuboids? Write it beside them.

Be familiar with similar and congruent shapes.

Example 22 Colour **similar** shapes in the same colour.

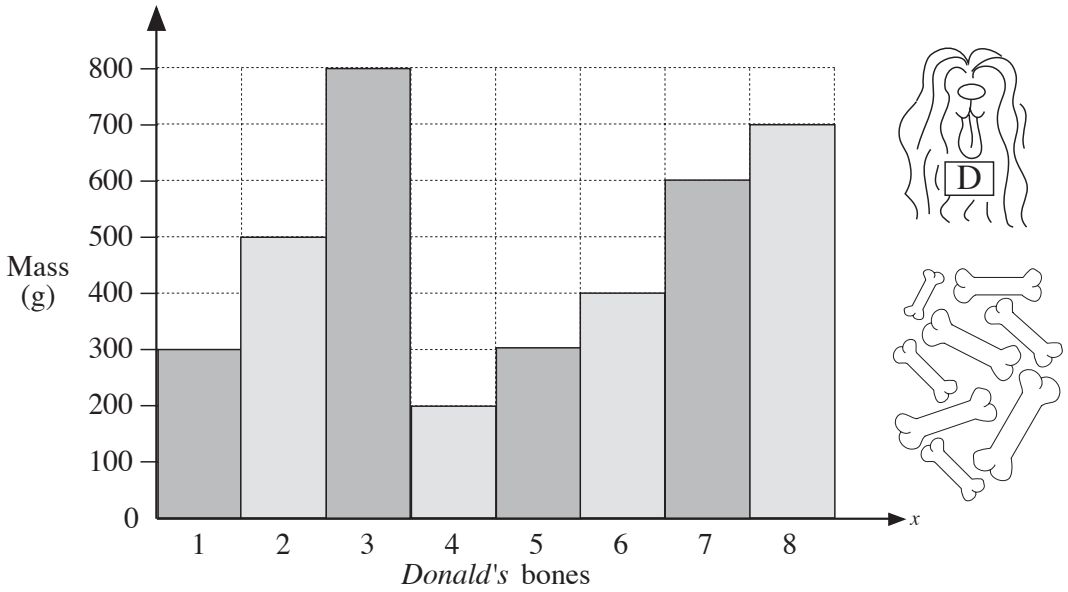


Example 23 Join up the shapes which are **congruent** (exactly the same).



Use bar charts and pictograms to illustrate data.

Example 24 *Donald Dog* was practising weighing. He numbered all his bones and weighed each one. Then he made this graph.



a) Which bone was: i) heaviest . . . . . ii) lightest? . . . . . .

b) Which two bones weighed the same? . . . . . .

c) Write the data from the graph in this table.

Bone number	1	2	3	4	5	6	7	8
Mass (g)								

Recognise the need for brackets in complex calculations.

Example 25 Do the calculations in the correct order.

a) $60 \div 6 + 4 \times 2 - 2 =$

b) $60 \div 6 + 4 \times (2 - 2) =$

c) $60 \div (6 + 4) \times 2 - 2 =$

d) $(60 \div 6 + 4) \times 2 - 2 =$

e) $60 \div (6 + 4 \times 2) - 2 =$

f) $60 \div (6 + 4) \times (2 - 2) =$

Example 26 Write the calculation **without** brackets so that the result is the same.

a) $128 + (30 + 5) =$ ☐ . . . . . .

b) $127 - (50 + 1) =$ ☐ . . . . . .

c) $146 - (90 - 16) =$ ☐ . . . . . .

d) $(50 - 7) \times 3 =$ ☐ . . . . . .

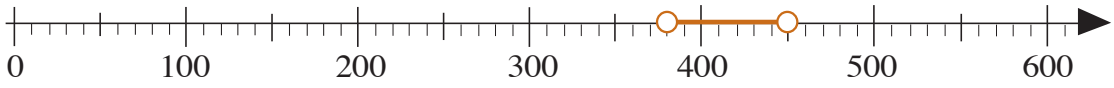
e) $(160 + 8) \div 8 =$ ☐ . . . . . .

LEARNING OBJECTIVES EXAMPLES: Answers

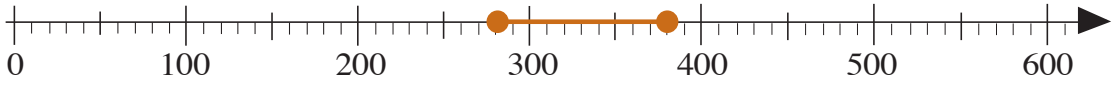
Example 1

Which whole numbers make each statement true? Mark them on the number line. Write down the highest and lowest possible numbers.

a) $380 < \square < 450$: 381 to 449



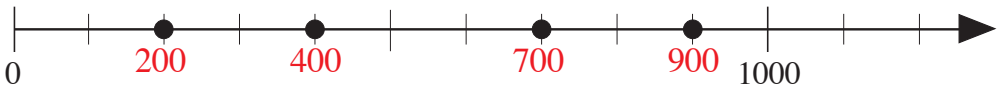
b) $280 \leq \square \leq 380$: 280 to 380



Example 2

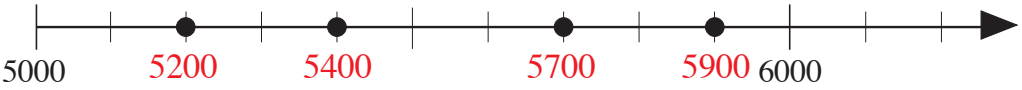
Write the numbers below the dots.

a)



b)

c)



Example 3

List all the 3-digit numbers in which:

a) the sum of the 3 digits is 5, (15 numbers)
113, 131, 311; 104, 140, 401, 410; 122, 212, 221; 203, 230, 302, 320; 500

b) the product of the 3 digits is 4, (6 numbers)
114, 141, 411, 122, 212, 221

c) the sum of the 3 digits is 4. (10 numbers)
103, 130, 301, 310; 112, 121, 211; 202, 220, 400

Example 4

Fill in the missing products. Note how they change.

a) $60 \times 3 = \boxed{180}$ $60 \times 6 = \boxed{360}$ $60 \times 9 = \boxed{540}$ $60 \times 12 = \boxed{720}$

b) $40 \times 5 = \boxed{200}$ $40 \times 10 = \boxed{400}$ $40 \times 15 = \boxed{600}$ $40 \times 25 = \boxed{1000}$

c) $4 \times 2 = \boxed{8}$ $40 \times 2 = \boxed{80}$ $400 \times 2 = \boxed{800}$ $40 \times 20 = \boxed{800}$

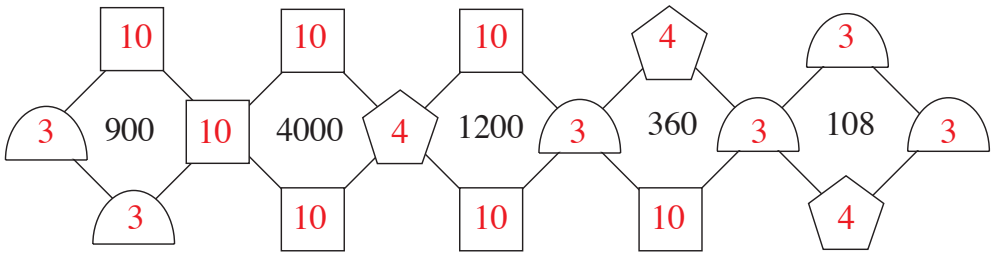
d) $3 \times 5 =$ 15 $30 \times 5 =$ 150 $300 \times 5 =$ 1500 $30 \times 50 =$ 1500

e) $4 \times 24 =$ 96 $8 \times 12 =$ 96 $16 \times 6 =$ 96 $2 \times 48 =$ 96

$4 \times 240 =$ 960 $8 \times 120 =$ 960 $16 \times 60 =$ 960 $2 \times 480 =$ 960

Example 5

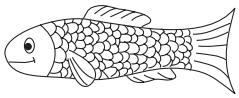
The middle number is the product of the 4 numbers around it. Fill in the missing numbers.



Example 6

Sparrow and *Trout* were arguing over the times in a day. Who is correct?

Tick the correct answer and cross out the wrong one.



✓ 12 hours	half a day	~~30 hours~~
~~14 hours~~	2 quarters of a day	12 hours ✓
✓ 4 hours	1 sixth of a day	4 hours ✓
~~45 minutes~~	2 half hours	60 minutes ✓
✓ 15 minutes	a quarter of an hour	~~20 minutes~~
✓ 40 minutes	2 thirds of an hour	~~45 minutes~~
~~2 hours~~	1 eighth of a day	3 hours ✓
~~9 hours~~	2 sixths of a day	8 hours ✓
✓ 18 minutes	3 tenths of an hour	~~20 minutes~~

Example 7

Fill in the missing numbers. ('min' means 'minutes' and 'hrs' means 'hours')

a) half an hour = 30 min

3 quarters of an hour = 45 min

3 fifths of an hour = 36 min

2 thirds of an hour = 40 min

5 sixths of an hour = 50 min

3 tenths of an hour = 18 min

2 and a half hours = 150 min

b) half a day = 12 hrs

2 thirds of a day = 16 hrs

3 quarters of a day = 18 hrs

5 eighths of a day = 15 hrs

1 twelfth of a day = 2 hrs

1 and a half days = 36 hrs

5 half days = 60 hrs

Example 8

Wendy went to Austria for a winter holiday. One day, she decided to note down the outside temperature every hour. She made this table to show her data.

Time (hours)	7	8	9	10	11	12	13	14	15	16	17	18	19
Temperature (°C)	– 9	– 10	– 6	– 2	0	3	6	8	9	7	4	– 1	– 3

a) When was it: i) coldest 8.00 am or 08.00 hours ii) warmest? 3.00 pm or 15.00 hours

b) Write the temperatures in increasing order.

$-10 < -9 < -6 < -3 < -2 < -1 < 0 < 3 < 4 < 6 < 7 < 8 < 9$

Example 9

Are the inequalities correct? Mark with a 4 or a 5. Correct the mistakes.

a) $-8 < -2$ ✓ b) $-20 \not> -10$ ✗ ($<$) c) $-5 < 5$ ✓ d) $-6 > -7$ ✓

e) $-10 < -9$ ✓ f) $-15 > -20$ ✓ g) $0 \not< -1$ ✗ ($>$) h) $-50 < 2$ ✓

Example 10

Two different numbers round to 300 as the nearest hundred. Is it possible that:

a) both numbers are less than 300 Yes

b) the smaller number is 100 less than the other number No

c) one number has 5 and the other has 0 as the tens digits Yes

d) both numbers are whole hundreds? No

Example 11

a) Complete the table.

Number	Rounded to nearest 10	Rounded to nearest 100
943	940	900
304	300	300
184	180	200
765	770	800
125	130	100
550	550	600
247	250	200
805	810	800

b) List all possible whole numbers which have:

- 5 as the tens digit when rounded to the nearest ten,

and also

- 5 as the hundreds digit when rounded to the nearest hundred.

3 digit numbers are:

450, 451, 452, 453, 454

545, 546, 547, 548, 549

Example 12

Fill in the missing numbers.

a) 1 m 30 cm = [130] cm b) 1 m 50 cm = [150] cm

c) 1 m 100 cm = [2] m = [200] cm

d) 1 m 26 cm = [126] cm e) 1 m 80 cm = [180] cm

f) 1 m 7 cm = [107] cm

Example 13

Fill in the missing numbers and units.

a) 1 kg = 1000 [g] b) half a kg = [500] g

c) ...one and a half... kg = 1500 g d) ....quarter of a. kg = 250 g

e) 1 tonne = 1000 [kg] f) half a tonne = [500] kg

Example 14

Fill in the missing numbers and units.

a) 2 litres = 200 [cl] b) 5 litres = [500] cl

c) 9 litres = [900] cl d) 3 litres 50 cl = 350 [cl]

e) 2 and a half litres = [250] cl f) 40 cl = [400] ml

Example 15

Complete the table.

ml	1200	2000	800	1230	1500	1900	1850
cl	120	200	80	123	150	190	185
10 cl	12	20	8	12 cl 3 ml	15	19	18 cl 5 ml
litres	1 and 2 tenths	2	8 tenths	1 and 23 hundredths	1 and 5 tenths	1 and 9 tenths	1 and 85 hundredths

---

Example 16

Is it possible to answer the question with the data given? If it is, solve it.

a) 10 kg of bananas costs £9.40. What is the price of 1 kg of bananas?

£9.40 = 940 p; 940 ÷ 10 = 94 p 1 kg of bananas costs 94 p.

b) Steve bought 10 different bars of chocolate and paid £12.00 altogether.

What was the price of 1 bar of chocolate?

Cannot be solved. Different bars might have different prices.

c) Karen is 9 years old. She weighs 27 kg.

What did she weigh when she was 1 year old?

There is no direct proportion between age and mass.

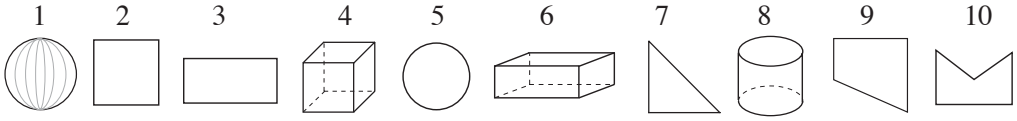
d) 3 men worked steadily and painted a 540 m fence in 9 days.

How many days would it have taken 1 man to paint the same fence?

3 men → 9 days, 1 man → 9 days × 3 = 27 days

Example 17

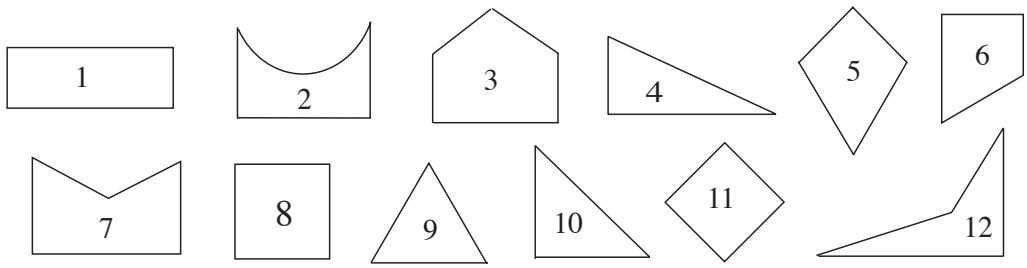
Which shape belongs in which box? Write the numbers in the correct boxes.



Plane shapes	Rectangles	Solids	Quadrilaterals
2, 3, 5, 7, 9, 10	2, 3	1, 4, 6, 8	2, 3, 9

Example 18

These **plane** shapes were cut out from coloured paper.



List the numbers of the shapes which are:

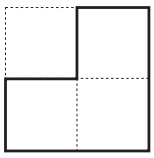
a) quadrilaterals: 1, 5, 6, 8, 11, 12

b) rectangles: 1, 8, 11

c) squares: 8, 11

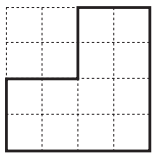
Example 19

Count how many of the given units are in the perimeter and area of each shape.

a) 

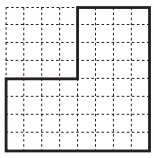
P = 8 ⊢⊣

A = 3 □

b) 

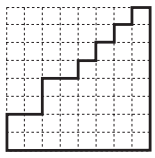
P = 16 ⊢⊣

A = 12 □

c) 

P = 32 ⊢⊣

A = 48 □

d) 

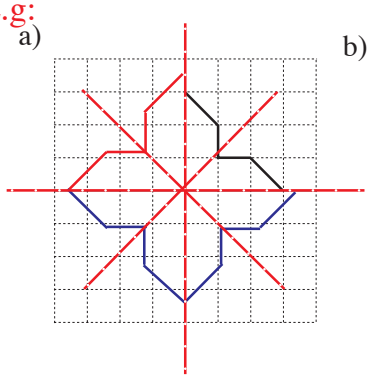
P = 32 ⊢⊣

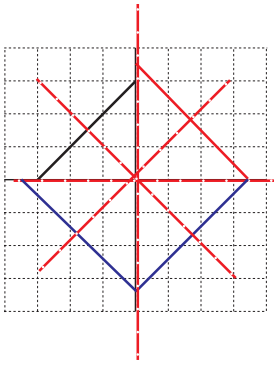
A = 38 □

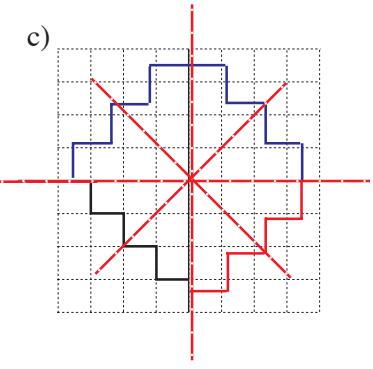
Example 20

Reflect the shape in one **axis** first. Then reflect the shape and its mirror image in the other **axis**. Draw the mirror lines of the whole shape.

E.g:

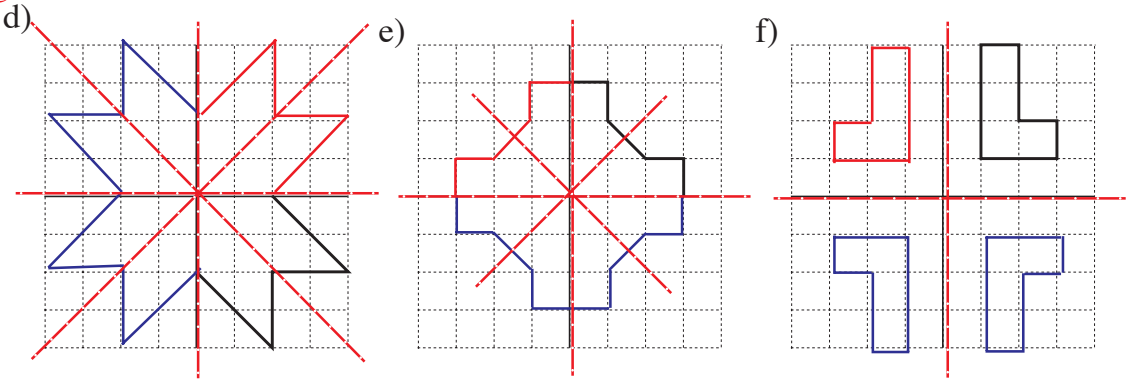
a) 

b) 

c) 

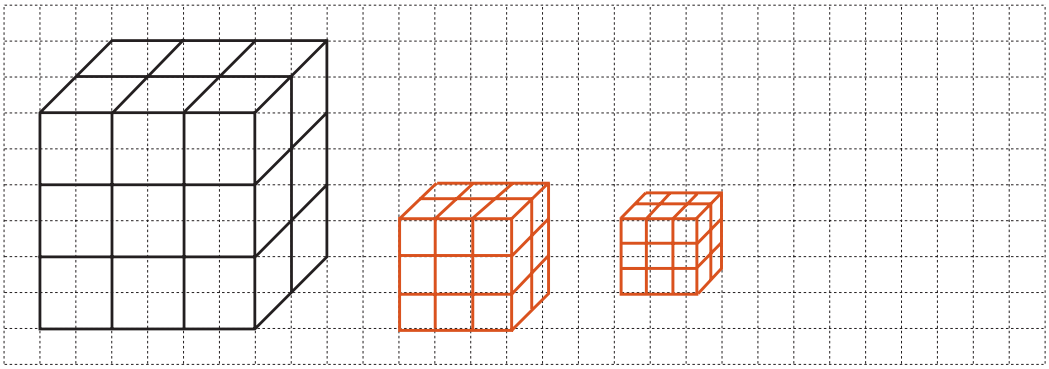
E.g:

d) e) f)

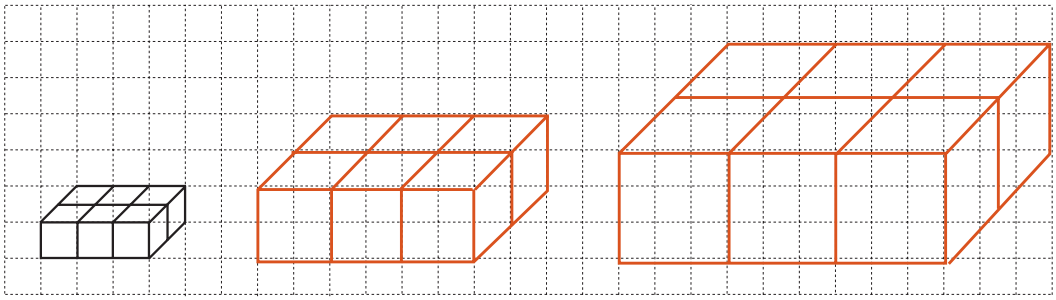


Example 21

a) Reduce this cuboid to: i) half its size ii) 1 third of its size.



b) Enlarge this cuboid to: i) twice its size ii) 3 times its size.

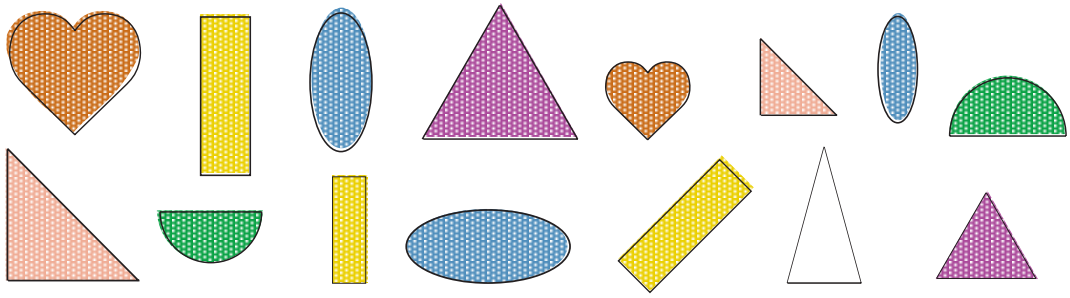


c) What is the volume of each of the 6 cuboids? Write it beside them.

$V = 6$ (unit cubes) $V = 48$ (unit cubes) $V = 162$ (unit cubes)

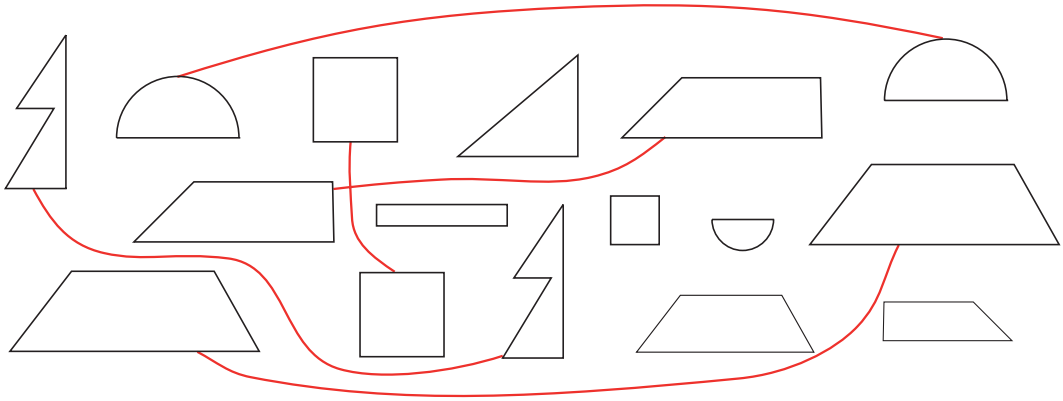
Example 22

Colour **similar** shapes in the same colour.



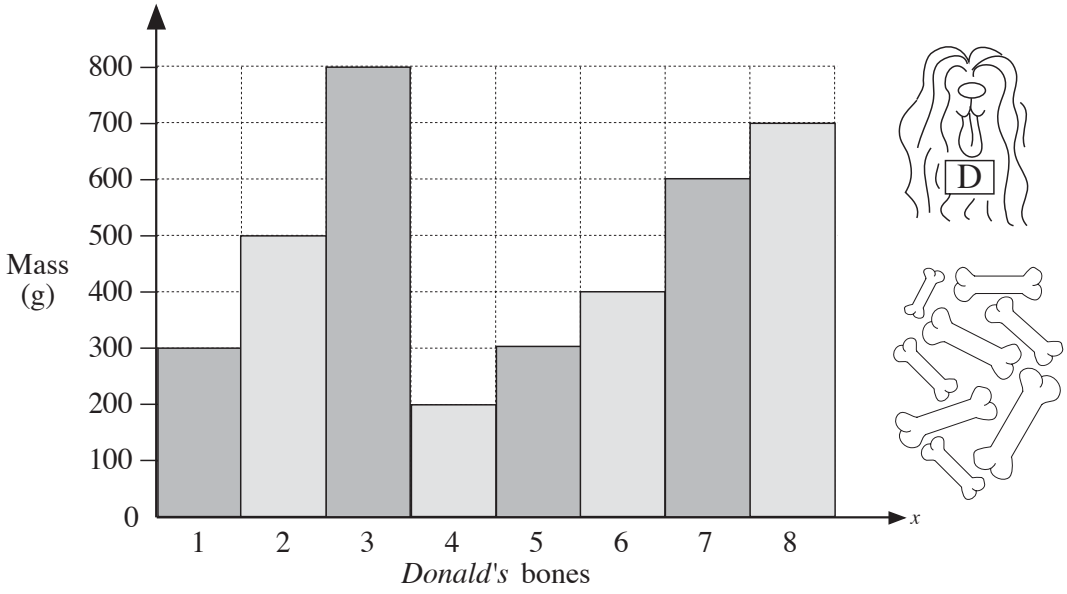
Example 23

Join up the shapes which are **congruent**. (exactly the same)



Example 24

Donald Dog was practising weighing. He numbered all his bones and weighed each one. Then he made this graph.



a) Which bone was: i) heaviest . . . . 3 . . . . . ii) lightest? . . . 4 . . . . . .

b) Which two bones weighed the same? . . . . . 1 and 5 . . . . . .

c) Write the data from the graph in this table.

Bone number	1	2	3	4	5	6	7	8
Mass (g)	300	500	800	200	300	400	600	700

Example 25

Do the calculations in the correct order.

a) $60 \div 6 + 4 \times 2 - 2 = 10 + 8 - 2 = 16$

b) $60 \div 6 + 4 \times (2 - 2) = 60 \div 6 + 4 \times 0 = 10 + 0 = 10$

c) $60 \div (6 + 4) \times 2 - 2 = 60 \div 10 \times 2 - 2 = 6 \times 2 - 2 = 12 - 2 = 10$

d) $(60 \div 6 + 4) \times 2 - 2 = (10 + 4) \times 2 - 2 = 14 \times 2 - 2 = 28 - 2 = 26$

e) $60 \div (6 + 4 \times 2 - 2) = 60 \div 12 = 5$

f) $60 \div (6 + 4) \times (2 - 2) = 60 \div 10 \times 0 = 6 \times 0 = 0$

Example 26

Write the calculation **without** brackets so that the result is the same.

a) $128 + (30 + 5) = \boxed{163}$ $128 + 30 + 5 = 163$

b) $127 - (50 + 1) = \boxed{76}$ $127 - 50 - 1 = 76$

c) $146 - (90 - 16) = \boxed{72}$ $146 - 90 + 16 = 72$

d) $(50 - 7) \times 3 = \boxed{129}$ $50 \times 3 - 7 \times 3 = 129$

e) $(160 + 8) \div 8 = \boxed{21}$ $160 \div 8 + 8 \div 8 = 21$

PROBLEM SOLVING

1. Write the operations **without** brackets if possible so that the result is the same.

Do the calculations as a check.

a) $(2 + 8) \times 7$ = =

b) $(11 - 3) \times 9$ = =

c) $(21 + 14) \div 7$ = =

d) $(24 - 8) \div 4$ = =

e) $80 \div (12 - 4)$ = =

f) $72 \div (3 + 6)$ = =

2. Fill in the missing numbers so that the equations are true, both horizontally and vertically.

	×		÷		= 4
×		÷		×	
	×		×		= 18
×		×		÷	
	×		÷		= 6
= 27		= 16		= 9	

3. I thought of a number. I divided it by 7 and the result was 8, remainder 6.

What is the number I was thinking of?

Calculation:

Check: *Answer:*

4. Which different 1-digit numbers could a, b and c be if $a + b + c = 14$ and $a \times b \times c = 84$?

5. How many different results can you find? Use +, –, or × signs.

70 ☐ 10 ☐ 3 = ☐ 70 ☐ 10 ☐ 3 = ☐

70 ☐ 10 ☐ 3 = ☐ 70 ☐ 10 ☐ 3 = ☐

70 ☐ 10 ☐ 3 = ☐ 70 ☐ 10 ☐ 3 = ☐

70 ☐ 10 ☐ 3 = ☐ 70 ☐ 10 ☐ 3 = ☐

70 ☐ 10 ☐ 3 = ☐ 70 ☐ 10 ☐ 3 = ☐

6. Two different numbers can be **rounded** to 70 as the nearest whole ten.

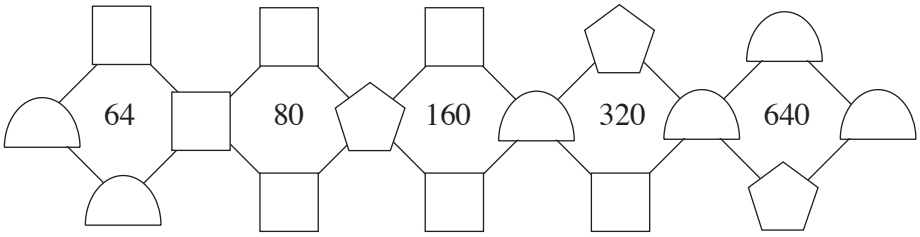
a) Is it possible that both numbers are less than 70?

b) Is it possible that one of the numbers is 10 less than the other?

c) Is it possible that one of them has 5 and the other has 0 as the units digits?

d) Is it possible that both numbers are whole tens?

7. The middle number is the **product** of the 4 numbers around it.



Fill in the missing numbers.

8. Create as many different 3-digit numbers as you can from the digits 1, 2, 3 and 4.
Do not use a digit more than once in any number.

9. Continue the sequences.

a) 1, 2, 4, 8, 16, . . .

b) 1, 4, 9, 16, 25, . . .

c) 0, 1, 1, 2, 3, 5, 8, . . .

d) 1, 3, 6, 10, 15, . . .

10. Fill in the missing digits.

a) i)

	_	_	_	
+	1	2	4	3
1	5	6	8	

ii)

	_	_	_	
+	9	1	3	
1	0	4	8	

iii)

	5	3	_	
+	_	_	1	
_	3	4	0	

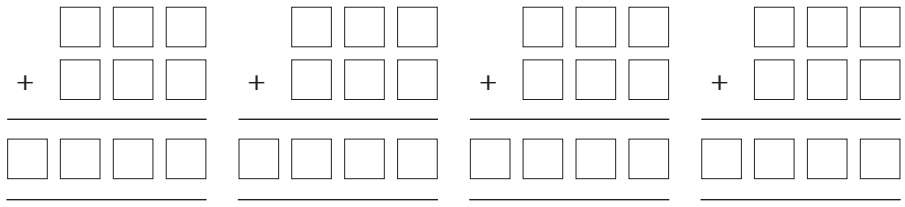
iv)

	5	_	7	
+	1	_	8	_
_	6	_	5	

v)

	9	7	_	
+	_	_	1	
_	3	3	_	

b) Write an addition which uses each of the digits from 0 to 9 once only.
Try out different solutions. Use your exercise books if you need to.



11. Which is more? How many more? Write subtractions and inequalities.

a) The smallest 4-digit number compared with the greatest 3-digit number.

b) The smallest 4-digit number compared with the smallest 3-digit number.

c) The smallest 4-digit number compared with the smallest 2-digit number.

d) The greatest 3-digit whole ten compared with the greatest 3-digit hundred.

e) The smallest 4-digit hundred compared with the smallest 4-digit whole ten.

f) The smallest whole hundred compared with the smallest whole ten.

12. Use every number on a dice only once in each subtraction, so that the subtraction makes sense and the difference is:

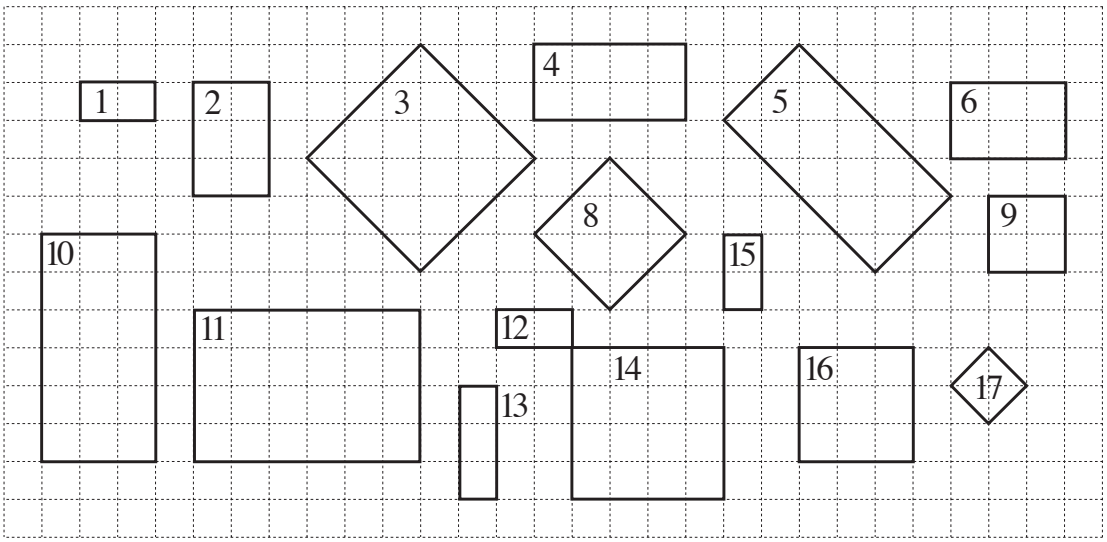
a) at least 300 b) the smallest possible c) between 200 and 300

d) even e) the greatest possible f) divisible by 10

13. Colour in the same colour shapes which are **similar** to

i) rectangle 1 ii) rectangle 2 iii) rectangle 3.

Use a different colour for each set of shapes.



14. Write these numbers in the correct place in the diagrams.

0, 4, 13, 30, 72, 95, 100, 321, 679, 1000, 1006, 1027, 2000

a)

Even	Odd

b)

Whole tens	Not whole tens

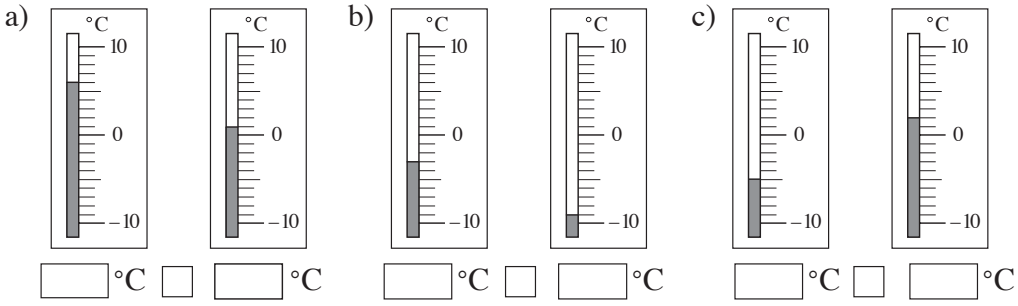
c)

3-digit	Not 3-digit

d)

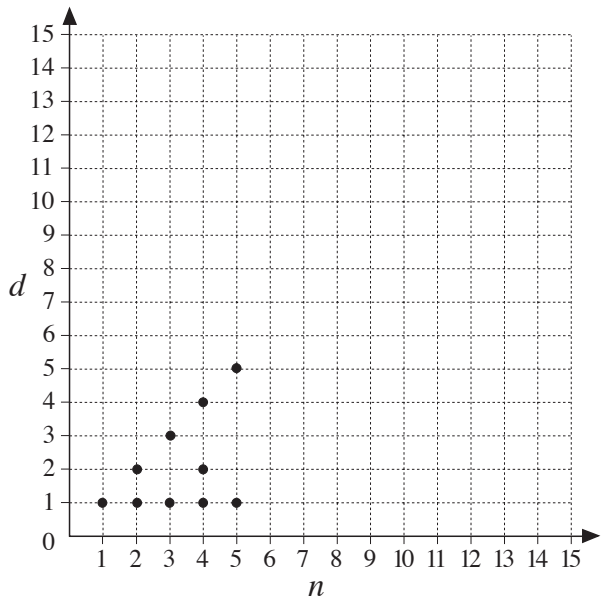
Whole hundreds	Not whole hundreds

15. Write the temperature below the thermometers. Write in the missing sign.

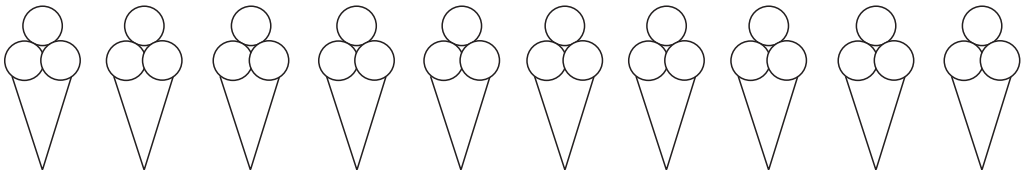


16. What is the rule? Complete the table and the graph.

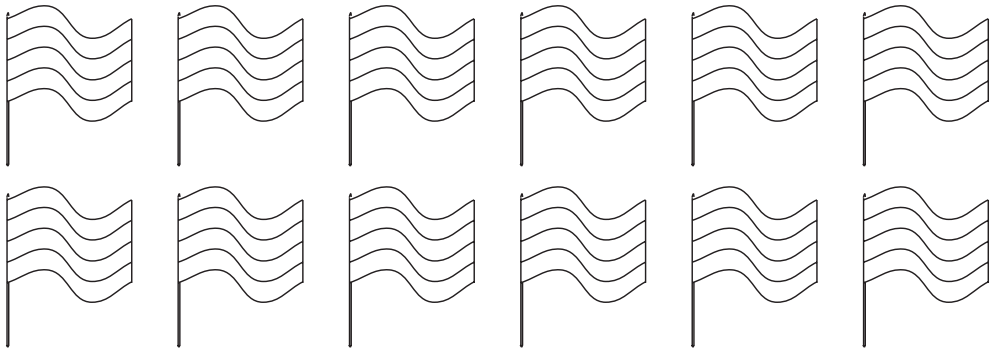
n	*d*
1	1
2	1, 2
3	1, 3
4	1, 2, 4
5	1, 5
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	



17. How could a 3-scoop ice-cream be made from vanilla or strawberry or lemon?



18. In how many different ways can you colour the flags *red*, *white*, *green* and *blue*?
Use every colour only once in each flag.



How many different ways are possible?

19. a) List in increasing order all the 3-digit numbers which have digits 1 or 2.

b) List in decreasing order all the 2-digit numbers which have digits 1, 2 or 3.

20. Make two 3-digit numbers using the numbers 0, 1, 3, 4, 5 and 8 so that:

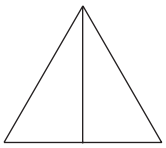
a) their sum is the least possible, ☐ and ☐

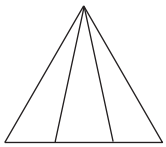
b) their sum is the greatest possible, ☐ and ☐

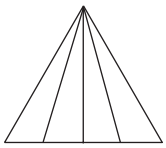
c) their difference is the least possible, ☐ and ☐

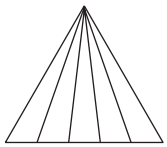
d) their difference is the greatest possible. ☐ and ☐

21. How many triangles can you see in each diagram?

a) 

b) 

c) 

d) 

PROBLEM SOLVING - SOLUTIONS

1. *Question and Solution*

Write the operations **without** brackets if possible so that the result is the same.
Do the calculations as a check.

a) $(2 + 8) \times 7 = 2 \times 7 + 8 \times 7 = 70$

b) $(11 - 3) \times 9 = 11 \times 9 - 3 \times 9 = 72$

c) $(21 + 14) \div 7 = 21 \div 7 + 14 \div 7 = 5$

d) $(24 - 8) \div 4 = 24 \div 4 - 8 \div 4 = 4$

e) $80 \div (12 - 4) =$ Not possible $= 10$

f) $72 \div (3 + 6) = 72 \div 3 + 72 \div 6 = 8$

Notes

The first four questions, a) to d), are straightforward; for example,

d) $(24 - 8) \div 4 = \boxed{24 \div 4 - 8 \div 4} = 6 - 2 = 4$

This is correct as, if we first calculate $24 - 8$, we get

$$(24 - 8) \div 4 = 16 \div 4 = 4$$

Parts e) and f) cannot be written with brackets in this way.

This is easy to see in f), where

$$72 \div (3 + 6) = 72 \div 9 = 8$$

but

$72 \div (3 + 6)$ is not equal to $72 \div 3 + 72 \div 6$ $(= 24 + 12 = 36)$

Learners will be taught later on about the BODMAS (or BIDMAS) convention which states that in calculations, the order used to complete operations is

- **Brackets**
- **Order** (powers or roots, e.g. $2^2 = 2 \times 2$) *or* **Indices**
- **Divide** or **Multiply**
- **Add** or **Subtract**

2. *Question and Solution*

Fill in the missing numbers so that the equations are true, both horizontally and vertically.

3	×	12	÷	9	= 4
×		÷		×	
3	×	3	×	2	= 18
×		×		÷	
3	×	4	÷	2	= 6
= 27		= 16		= 9	

Notes

At this stage, we expect learners to use some known factor facts; for example,

$$27 = 1 \times 1 \times 27 \quad = 1 \times 3 \times 9 \quad = 3 \times 3 \times 3$$

$$18 = 1 \times 1 \times 18 \quad = 1 \times 2 \times 9 \quad = 3 \times 2 \times 3 \quad = 1 \times 3 \times 6$$

As the column for 27 (first column) and the row for 18 (second row) have one number in common, it could be 1, 3 or 9.

There are in fact a number of possible answers, but you could add in the constraint that,

"1 is not allowed as a factor."

This would mean that the first column would be $3 \times 3 \times 3$ and the third row would be $3 \times 2 \times 3$ or $3 \times 3 \times 2$.

Using trial and error now gives the two possible answers:

3	X	8	÷	6	= 4
X		÷		X	
3	X	2	X	3	= 18
X		X		÷	
3	X	4	÷	2	= 6
= 27		= 16		= 9	

3	X	12	÷	9	= 4
X		÷		X	
3	X	3	X	2	= 18
X		X		÷	
3	X	4	÷	2	= 6
= 27		= 16		= 9	

---

3. *Question and Solution*

I thought of a number. I divided it by 7 and the result was 8, remainder 6.

What is the number I was thinking of?

Calculation: $7 \times 8 + 6 = 62$

Check: $62 \div 7 = 8$, remainder 6 *Answer:* 62

Notes

The straightforward way here is to call the missing number x.
Then the calculation is

$$x \div 7 = 8 \text{ remainder } 6$$

So

$$x = 7 \times 8 + 6$$

$$= 56 + 6$$

$$= 62$$

---

4. *Question and Solution*

Which different 1-digit numbers could a, b and c be if $a + b + c = 14$ and $a \times b \times c = 84$?

$a =$ 3 $b =$ 4 $c =$ 7

Notes

One approach would be to consider values of a, then values of b and c for each value of a; for example,

			$a \times b \times c$	
$a = 1$	$\Rightarrow$ $b + c = 13$	$\Rightarrow$ $b = 4,\ c = 9$	$1 \times 4 \times 9 = 36$	X
		$b = 5,\ c = 8$	$1 \times 5 \times 8 = 40$	X
		$b = 6,\ c = 7$	$1 \times 6 \times 7 = 42$	X
		$b = 7,\ c = 6$	$1 \times 7 \times 6 = 42$	X
		$b = 8,\ c = 5$	$1 \times 8 \times 5 = 40$	X
		$b = 9,\ c = 4$	$1 \times 9 \times 4 = 36$	X

So there are no solutions with $a = 1$.

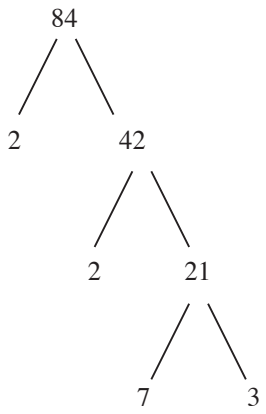
In a similar way, $a = 2$ gives no solutions, but when

$a = 3$	$\Rightarrow$ $b + c = 11$	$\Rightarrow$ $b = 2,\ c = 9$	$3 \times 2 \times 9 = 54$	X
		$b = 3,\ c = 8$	$3 \times 3 \times 8 = 72$	X
		$b = 4,\ c = 7$	$3 \times 4 \times 7 = 84$	✓

So one solution is $a = 3$, $b = 4$ and $c = 7$.

To find any more answers, we would have to continue in this way.

A more efficient approach would be to find the factors of 84.



So $84 = 2 \times 2 \times 7 \times 3$ and this gives the possible products

$$4 \times 7 \times 3 \quad \text{or} \quad 2 \times 7 \times 6$$

but $2 + 7 + 6 = 15$, i.e. not 14, and hence not a solution.

Hence there are six possible solutions all based on $4 \times 7 \times 3$:

a	*b*	*c*	
4	7	3	based on $4 \times 7 \times 3$
4	3	7	
7	3	4	
7	4	3	
3	7	4	
4	7	3	

---

5. *Question and Solution*

How many different results can you find? Use +, –, or × signs.

70	+	10	+	3	=	83	70	×	10	−	3	=	697
70	+	10	−	3	=	77	70	+	10	×	3	=	100
70	−	10	+	3	=	63	70	−	10	×	3	=	40
70	−	10	−	3	=	57	70	×	10	×	3	=	2100
70	×	10	+	3	=	703	70		10		3	=	

Notes

Note that, with three signs there will be only 9 possibilities (so one of the options remains blank).

The conventional method is to use BODMAS:

$$70 + (10 \times 3) = 70 + 30 = 100$$

but if learners haven't yet met this, calculating from the left hand side will give

$$70 + 10 = 80 \text{ and } 80 \times 3 = 240$$

---

6. *Question and Solution*

Two different numbers can be **rounded** to 70 as the nearest whole ten.

a) Is it possible that both numbers are less than 70? Yes (e.g. 65, 66)

b) Is it possible that one of the numbers is 10 less than the other? No

c) Is it possible that one of them has 5 and the other has 0 as the units digits? Yes (65 and 70)

d) Is it possible that both numbers are whole tens? No

Notes

a) As these numbers all round to 70 to the nearest 10, any two of them would be a correct answer.

b) The full list of numbers is

65, 66, 67, 68, 69, 70, 71, 72, 73 and 74

so the greatest difference is $74 - 65 = 9$.

c) 65 and 70 is the only solution.

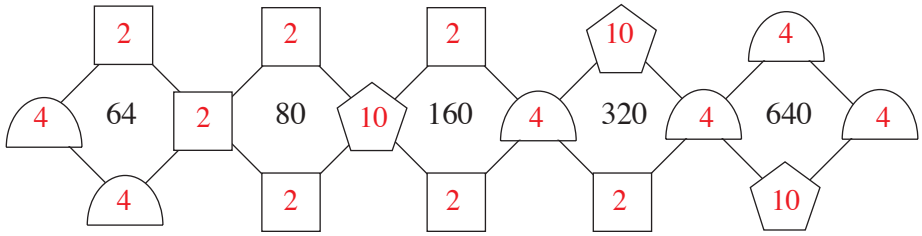
d) Only 70 is a whole ten.

Note that it is a convention that 65 rounds up to 70, to the nearest 10. 65 is in fact exactly half way between 60 and 70. This 'round up the middle number' rule applies for all rounding of numbers.

---

7. *Question and Solution*

The middle number is the **product** of the 4 numbers around it.



Notes

It is tempting to guess a few numbers and see if they work, but the easiest *logical* method is to consider the last set of four numbers. Here

$640 = a \times a \times a \times b$ where $a =$ ⌓ and $b =$ ⬠

Now, factorising,

$$\begin{aligned} 640 &= 64 \times 10 \\ &= 8 \times 8 \times 10 \\ &= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 10) \end{aligned}$$

and to put it in the form above, we could have

$640 = 4 \times 4 \times 4 \times 10$ (A)

or

$640 = 2 \times 2 \times 2 \times 80$ (B)

or

$640 = 1 \times 1 \times 1 \times 640$ (C)

It is clear that (C) does not work as having ⬠ $= 640$ would contradict the four numbers around 80 or 160.

Now consider (B); here ⬠ $= 80$ and ⌓ $= 2$. Working on the right hand set of numbers, we would have

$2 \times 2 \times \square \times \square = 64$

or

$$\square \times \square = 16$$

So

$$\square = 4$$

In the next set of 4 numbers,

$$4 \times 4 \times 4 = 80 \text{ should equal } 80$$

but this is not true. Hence (B) does not work.

Thus (C) is the correct solution and we can see that

$$\cap = 4,\ \pentagon = 10,\ \square = 2$$

satisfies each set of numbers in the diagram.

---

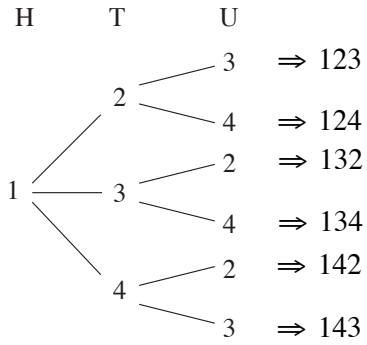
8. *Question and Solution*

Create as many different 3-digit numbers as you can from the digits 1, 2, 3 and 4. Do not use a digit more than once in any number.

123	234	124	134
213	243	142	143
312	324	214	314
132	342	241	341
231	432	412	413
321	423	421	431

Notes

In this question, we need to be logical and systematic. Start with the first digit:



So there are 6 3-digit numbers starting with 1.

Exactly the same holds for numbers starting with 2, 3 and 4. Hence there are

$$6 + 6 + 6 + 6 = 4 \times 6 = 24$$

different 3-digit numbers using each of the digits 1, 2, 3 and 4 not more than once.

---

9. *Question and Solution*

Continue the sequences.

a) 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, . . .

b) 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 . . .

c) 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . .

d) 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, . . .

Notes

For sequences, we need to find the 'rule' on which they are based. So here we have

a) 1, $2 = 2 \times 1$, $4 = 2 \times 2$, $8 = 2 \times 4$, $16 = 2 \times 8$

and the rule is

next number = 2 × previous number

Using this rule, we obtain

$$2 \times 16 = 32, \quad 2 \times 32 = 64, \quad 2 \times 64 = 128, \text{ etc.}$$

b) $1 = 1 \times 1$, $4 = 2 \times 2$, $9 = 3 \times 3$, $16 = 4 \times 4$, $25 = 5 \times 5$

The rule is

$n \times n$

when n is 1, 2, 3, 4, 5, 6, 7, ... (The product of a number which has been multiplied by itself is called a *square number*.)

Thus the next terms are

$$6 \times 6 = 36, \quad 7 \times 7 = 49, \quad 8 \times 8 = 64, \quad 9 \times 9 = 81, \quad 10 \times 10 = 100, \quad ...$$

c) This is not so straightforward. If we look carefully, though, we can see that

$$0 + 1 = 1$$
$$1 + 1 = 2$$
$$1 + 2 = 3$$
$$2 + 3 = 5$$
$$3 + 5 = 8$$

that is,

next number = sum of previous two numbers in the sequence

This is known as a *Fibonacci sequence*. There are numerous examples of Fibonacci sequences in nature including the breeding pattern for rabbits (and also mice) in ideal conditions.

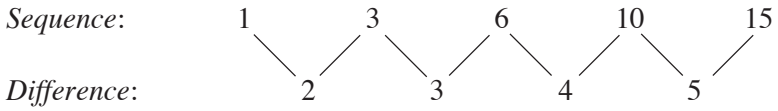
The next terms are

$$5 + 8 = 13, \quad 8 + 13 = 21, \quad 13 + 21 = 34, \quad \text{etc.}$$

Teaching point

The number of petals in a flower often follows the Fibonacci sequence (there are very few flowers with 4 petals; 4-leaf clovers are considered lucky as they are very rare!)

d) Again we need to look carefully and consider the difference in each term:



Now we can see that the rule is

differences increase by one

and hence the next terms are

15 + 6 = 21, 21 + 7 = 28, 28 + 8 = 36, etc.

---

10. *Question and Solution*

Fill in the missing digits.

a) i)

	3	2	5	
+	1	2	4	3
	1	5	6	8

ii)

	1	3	5	
+	9	1	3	
	1	0	4	8

iii)

	5	3	9	
+	8	0	1	
1	3	4	0	

iv)

	5	0	7	
+	1	1	8	8
1	6	9	5	

v) Possible answer

	9	7	8	
+	3	6	1	
1	3	3	9	

Notes

a) Each one of these can be deduced from the logic of calculation; for example,

iii)

	5	3	___	
+	___	___	1	
___	3	4	0	

First note that in the sum the THOUSAND digit can only be 1.

	5	3	___	
+	___	___	1	
1	3	4	0	

Working from the right hand side, the missing UNIT digit has to be 9.

	5	1 3	9	
+	___	___	1	
1	3	4	0	

Now we can see that the missing TENS digit has to be 0.

$$\begin{array}{rcccc} & & 5 & {}^{1}3 & \mathbf{9} \\ + & & \_\_ & \mathbf{0} & 1 \\ \hline & \mathbf{1} & 3 & 4 & 0 \\ \hline \end{array}$$

$$\begin{array}{rcccc} & {}^{1}0 & 5 & {}^{1}3 & 9 \\ + & \mathbf{8} & \mathbf{0} & 1 & \\ \hline & \mathbf{1} & 3 & 4 & 0 \\ \hline \end{array}$$

Finally, the HUNDREDS digit has to be 8 (as we need to carry 1 into the thousands column).

Note that in parts iv) and v) there are more possible correct answers than those given.

b) This is more of a challenge but able learners will not only enjoy this but will be able to find many solutions. For example, we can see that,

1. The missing THOUSAND digit in the sum has to be 1.
2. Check whether you can solve the puzzle with only the HUNDREDS digit 'crossing ten'; for example, $6 + 4 = 10$.
3. We could look now for combinations from the remaining digits, 2, 3, 5, 7, 8, 9, that do *not* cross 10; for example, $2 + 7 = 9$, $5 + 3 = 8$.
4. We now have many solutions; for example,

$$\begin{array}{rcccc} & & 6 & 2 & 5 \\ + & & 4 & 7 & 3 \\ \hline & 1 & 0 & 9 & 8 \\ \hline \end{array} \qquad \text{or} \qquad \begin{array}{rcccc} & & 6 & 2 & 3 \\ + & & 4 & 7 & 5 \\ \hline & 1 & 0 & 9 & 8 \\ \hline \end{array}$$

(How many solutions can you find that take this form?)

---

11. *Question and Solution*

Which is more? How many more? Write subtractions and inequalities.

a) The smallest 4-digit number compared with the greatest 3-digit number.

$1000 > 999$ $1000 - 999 = 1$

b) The smallest 4-digit number compared with the smallest 3-digit number.

$1000 > 100$ $1000 - 100 - 900$

c) The smallest 4-digit number compared with the smallest 2-digit number.

$1000 > 10$ $1000 - 10 = 990$

d) The greatest 3-digit whole ten compared with the greatest 3-digit hundred.

$990 > 900$ $990 - 900 = 90$

e) The smallest 4-digit hundred compared with the smallest 4-digit whole ten.

$1000 > 1000$ $1000 - 1000 = 0$

f) The smallest whole hundred compared with the smallest whole ten.

$100 > 10$ $100 - 10 = 90$

Notes

This tests a number of concepts, especially place value and understanding of greatest, least, etc.

a) 1000 – 999 = 1 1000 $\overset{1}{\rangle}$ 999

b) 1000 – 100 = 900 1000 $\overset{900}{\rangle}$ 100

c) 1000 – 10 = 990 1000 $\overset{990}{\rangle}$ 10

d) 990 – 900 = 90 990 $\overset{90}{\rangle}$ 900

e) 1000 – 1000 = 0 (careful with this one!)

f) 100 – 10 = 90 100 $\overset{90}{\rangle}$ 10

---

12. *Question and Solution*

Use every number on a dice only once in each subtraction, so that the subtraction makes sense and the difference is:

a) at least 300 b) the smallest possible c) between 200 and 300

e.g.

	6	5	4
–	2	3	1
	4	2	3

	4	1	2
–	3	6	5
		4	7

	6	5	3
–	4	1	2
	2	4	1

d) even e) the greatest possible f) divisible by 10

e.g.

	4	6	5
–	3	2	1
	1	4	4

	6	5	4
–	1	2	3
	3	3	1

–			

Impossible

Notes

Some of these questions have many answers, but care is needed. We use only the digits 1, 2, 3, 4, 5 and 6 in the first *two* rows, but any digits in the final row; for example,

e) Here 654 is the greatest number possible whilst 123 is the smallest, to give

	6	5	4
–	1	2	3
	5	3	1

f) Now we need the sum to end in a zero - but this cannot be possible! Hence no answer!

(Hungarians often set impossible questions in order to make learners think.)

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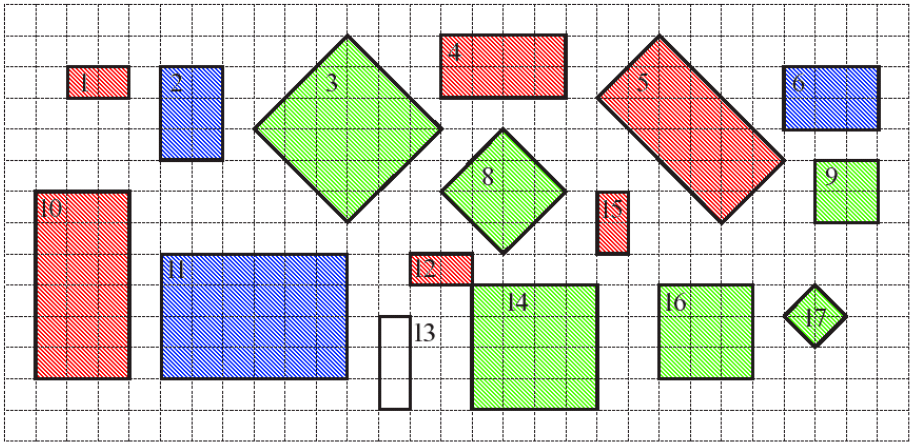
13. *Question and Solution*

Colour in the same colour shapes which are **similar** to

i) rectangle 1 ii) rectangle 2 iii) rectangle 3.

Use a different colour for each set of shapes.



Notes

Note that, for rectangles to be SIMILAR, the adjacent sides must be in the same ratio; for example,

In RECTANGLE 1, the adjacent sides are in the ratio 1 : 2 and in RECTANGLE 4, the ratio is 2 : 4 which is equivalent to 1 : 2.

Also, in RECTANGLE 5, the ratio is 2 : 4 (but using diagonal sides) and this is also 1 : 2.

(We often write this as $2 : 4 \equiv 1 : 2$, the sign '$\equiv$' meaning 'is equivalent to', but for Year 3 learners, it is sufficient to say 'equal to' or 'the same as'.)

The ratio of the sides in RECTANGLE 10 is $6 : 3 \equiv 2 : 1$. This is the same as 1 : 2 (but rotated by one right angle).

RECTANGLE 12 is identical to RECTANGLE 1; they are congruent and hence also similar.

Thus RECTANGLES 1, 4, 5, 10 and 12 are similar.

We can see that RECTANGLES 2 and 6 are similar (in fact, they are identical, with adjacent sides in the ratio 2 : 3) and RECTANGLE 11, with sides in the ratio $4 : 6 \equiv 2 : 3$ is also similar.

RECTANGLES 3, 8, 9, 14, 16 and 17 are similar (they are all squares).

RECTANGLE 11 (with sides in the ratio $4 : 6 \equiv 2 : 3$) and RECTANGLE 13 (with sides in the ratio 3 : 1) are not similar to any of the other rectangles.

14. *Question and Solution*

Write these numbers in the correct place in the diagrams.

0, 4, 13, 30, 72, 95, 100, 321, 679, 1000, 1006, 1027, 2000

a)

Even	Odd
0 4 30 72	13 95
100 1000	321 679
1006 2000	1027

b)

Whole tens	Not whole tens
0 30 100	4 13 72
1000	95 321 679
2000	1006 1027

c)

3-digit	Not 3-digit
100 321	0 4 13 72
679	95 1000
	1027 2000

d)

Whole hundreds	Not whole hundreds
0 100	4 13 30 72
1000	95 321 679
2000	1006 1027

Notes

This is a straightforward question on classification. You can revise or extend the question by asking learners to decide on their own method of classification (this is the Japanese way of using 'open approach' problem solving when there are many possible answers). In fact, you could give ownership to your learners by first asking each of them to produce their own set of 1, 2 or 3 digit numbers to classify!

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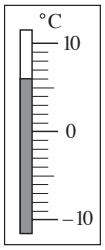
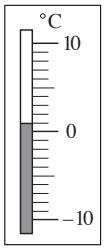
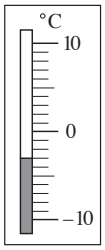
15. *Question and Solution*

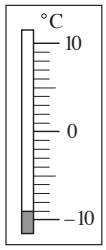
Write the temperature below the thermometers. Write in the missing sign.

a) 

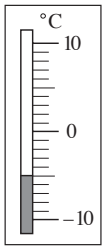


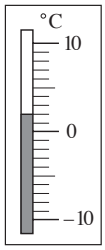
6 °C > 1 °C

b) 



−3 °C > −9 °C

c) 



−5 °C < 2 °C

Notes

This question focuses on negative numbers using temperature as the context. Your learners should not have difficulty identifying the values, that is,

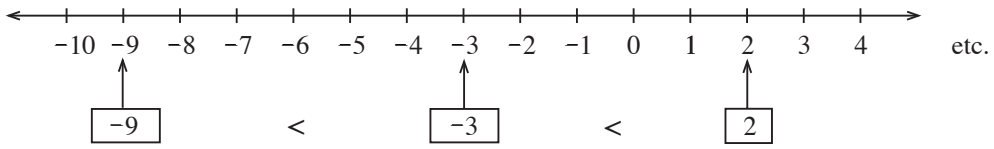
6, 1, −3, −9, −5, 2 (°C)

but might have more difficulty with the inequality signs between the values.

$6 > 1 > -3 > -9 < -5 < 2$

The $-3 > -9$ sometimes looks odd as $3 < 9$, but keep referring to the positions of the numbers on a number line.

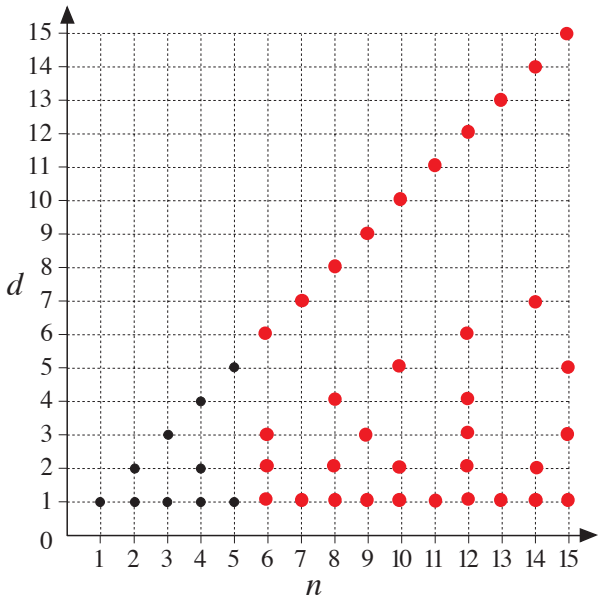
You might find it helpful to have a horizontal extended number line on the board.



16. *Question and Solution*

What is the rule? Complete the table and the graph.

n	d
1	1
2	1, 2
3	1, 3
4	1, 2, 4
5	1, 5
6	1, 2, 3, 6
7	1, 7
8	1, 2, 4, 8
9	1, 3, 9
10	1, 2, 5, 10
11	1, 11
12	1, 2, 3, 4, 6, 12
13	1, 13
14	1, 2, 7, 14
15	1, 3, 5, 15



Notes

This is a really useful question for illustrating a number of concepts. First, though, your learners must complete the table (which shows the divisors of each number) and the chart.

You could ask learners to spot the patterns and deduce that

- numbers divisible by 2 are in the '2' row
- numbers divisible by 3 are in the '3' row, etc.
- only one number, 1, has just one divisor.

Or ask

- which numbers have exactly 2 divisors? (These are prime numbers.)
- which numbers have exactly 3 divisors? (These are square numbers.)

An extension is to look for any numbers whose divisors, excluding the number itself, add up to itself. We can see that there is only one, that is 6 (as $1 + 2 + 3 = 6$). This is called a PERFECT number.

Are there any more perfect numbers?

(Not in $1 \rightarrow 15$, but 28 is the second perfect number.) A useful reference for perfect numbers is http://mathforum.org/dr.math/faq/faq.perfect.html

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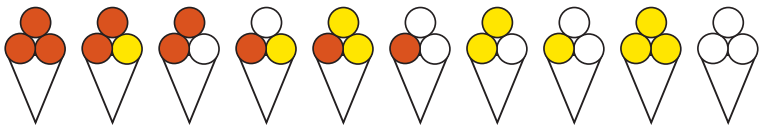
17. *Question and Solution*

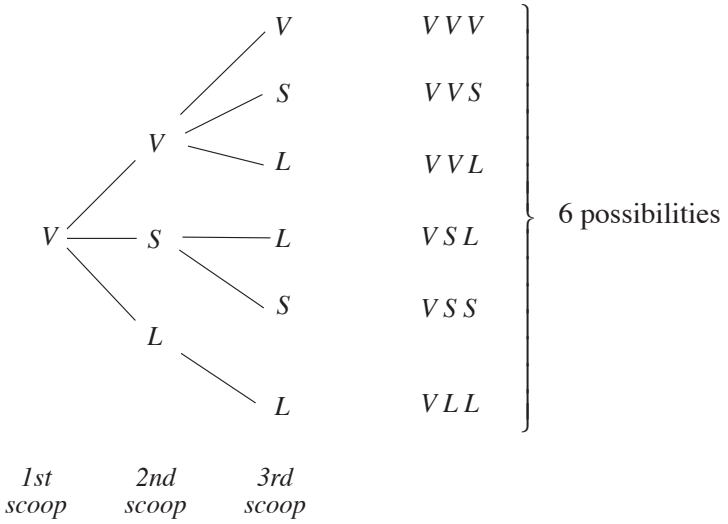
How could a 3-scoop ice-cream be made from vanilla or strawberry or lemon?



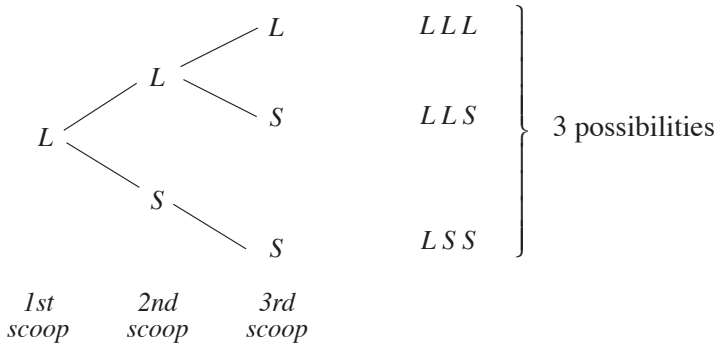
Notes

This is an example of a 'combinatorics' question, that is, a counting question. We look for efficient ways to answer such questions or at least systematic ways of counting.

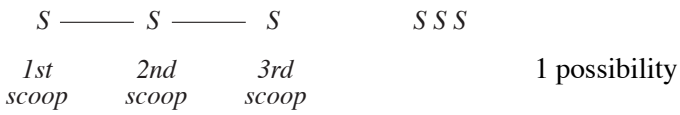
We first assume that the actual position of each scoop does not matter. Then, we could start by choosing vanilla (V) and could list the possible combinations, making sure that there are no repeats.



Now start with lemon (L) to give new possibilities.



Finally, starting with strawberry (S),



This gives $6 + 3 + 1 = 10$ possibilities.

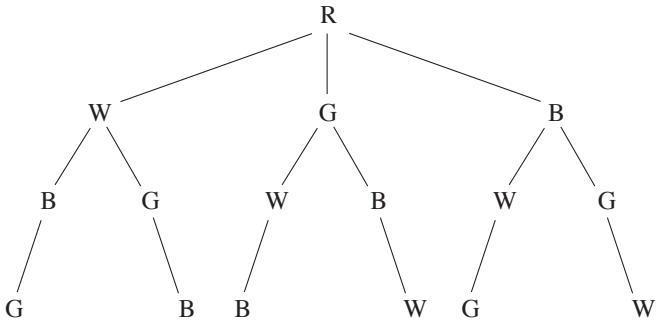
18. *Question and Solution*

In how many different ways can you colour the flags *red*, *white*, *green* and *blue*?
Use every colour only once in each flag.

There are 24 different ways (or 12, as the flags can be flown upside-down).

Notes

This is another combinatorics problem. Yes, we could start colouring in, but it is much more efficient to make a chart (R, W, G and B). Start with red (R).



So, starting with R, we have 6 possible flags,

R W B G	R G W B	R B W G
R W G B	R G B W	R B G W

The same arrangement can be used for starting with W (white), G (green) or B (blue), to give

$$6 + 6 + 6 + 6 = 4 \times 6 = 24 \text{ possibilities}$$

Why are there only 12 empty flags ishon in the question?

This is partly to encourage learners to think rather than rush into colouring and also as, for example,

R W B G

will also occur in the reverse colouring, G B W R.

So there are only 12 distinct colourings if you ignore the vertical direction of the flag!

An even quicker (and more mathematical) way of obtaining the answer is as follows:

For the top row there are 4 choices of colour

Having found the colour of the top row, there will be 3 choices of colour for row 2.

Having found the colours of the first 2 rows, there will be 2 choices of colour for row 3.

Having found the colours of the first 3 rows, there will be only 1 possible colour for row 4.

This gives

$$4 \times 3 \times 2 \times 1 = 24$$

possibilities. (This method is only for very, very able learners at this stage!)

19. *Question and Solution*

a) List in increasing order all the 3-digit numbers which have digits 1 or 2.

111, 112, 121, 122, 211, 212, 221, 222

b) List in decreasing order all the 2-digit numbers which have digits 1, 2 or 3.

33, 32, 31, 23, 22, 21, 13, 12, 11

Notes

Logical, systematic thinking is again required to give, for example:

a) 111, 112, 121, 122, 211, 212, 221, 222

---

20. *Question and Solution*

Make two 3-digit numbers using the numbers 0, 1, 3, 4, 5 and 8 so that:

a)	their sum is the least possible,	108	and	345
b)	their sum is the greatest possible,	841	and	530
c)	their difference is the least possible,	401	and	385
d)	their difference is the greatest possible.	854	and	103

Notes

Again, logical thinking is needed.

a) We need the two smallest numbers, so the HUNDREDS digits must be 1 and 3 (we cannot start the number with 0) and use 0 and 4 for the TENS and 5 and 8 for the UNITS.

For example,

105 and 348 to give sum = 453

(there are a number of possible solutions but they have the same sum,

i.e. 145 and 308

148 and 305

108 and 345)

b) This is the reverse of the problem in a), so we put 8 and 5 as the HUNDREDS, 4 and 3 as the TENS and 0 and 1 as the UNITS, to give, for example,

840 and 531 with sum = 1371

c) We need to make the numbers as close to one another as possible so the first digits must 3 and 4 or 4 and 5.

For 3 and 4, we have 385 (largest number) and 401 (smallest number) to give a difference of 16.

For 4 and 5, we have 483 (largest number) and 501 (smallest number) to give a difference of 18.

Hence we choose 401 and 385.

d) This is the reverse of the problem in c). We cannot start the number with 0, so the two numbers will start with 1 and 8, and we choose

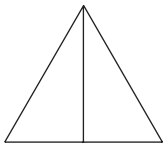
103 (smallest number) and 854 (largest number)

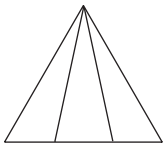
to give a difference of 751.

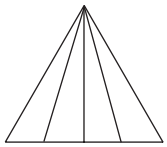
---

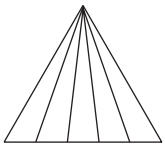
21. *Question and Solution*

How many triangles can you see in each diagram?

a) 

b) 

c) 

d) 

$2 + 1 = 3$ $3 + 2 + 1 = 6$ $4 + 3 + 2 + 1 = 10$ $5 + 4 + 3 + 2 + 1 = 15$

Notes

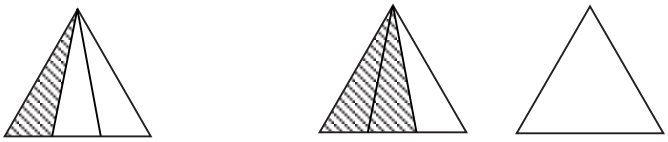
The important aspect of this problem is to be able to generalise.

For a), we clearly have

$$2\ (\text{half-size } \Delta) + 1\ (\text{large } \Delta) = 3\ \Delta \quad (= 2 + 1)$$

For b), we have

$$3\ (\text{one third-size } \Delta) + 2\ (\text{two third-size } \Delta) + 1\ (\text{large } \Delta)\ = 6\ \Delta \quad (= 3 + 3)$$



For c), we have

$$4\ (\text{quarter-size } \Delta) + 3\ (\text{half-size } \Delta) + 2\ (\text{three-quarter size } \Delta)1 + 1\ (\text{large } \Delta)\ = 10\ \Delta$$
$$(= 4 + 6)$$

For d), we have

$$5\ (\text{one fifth-size } \Delta) + 4\ (\text{two fifth-size } \Delta) + 3\ (\text{three fifth-size } \Delta) +$$
$$2\ (\text{four fifth-size } \Delta) + 1\ (\text{large } \Delta)\ = 15\ \Delta \quad (= 5 + 10)$$

So what about the next triangle? It will have $6 + 15 = 21\ \Delta$, etc.

Mathematics Enhancement Programme

TEACHING SUPPORT: Book 3

INTRODUCTION TO LESSON PLANS

Detailed Lesson Plans* have been developed to be used as guidance for *MEP Primary* teaching. They are based on

4 × 45 minute lessons per week for 35 weeks

The online Revision Questions are for you to use for learners' practice and revision, perhaps for the fifth lesson of the week or for homework.

The Lesson Plans have been adapted from the Eastern European approach to mathematics teaching. The first few weeks of the course provide introductory activities to help you assess the capabilities of your class and to bring them together at a suitable starting point for the systematic treatment.

The course provides a very strong mathematical foundation, dealing with topics in depth and in a logical sequence, and using appropriate notation. It provides the necessary background in mathematical logic, aiming to give enhanced attainment in national tests and a solid framework for higher level work.

The Lesson Plans make reference to:

- Practice Books (e.g. *Practice Book 3*)

and make use of a range of resources:

- number cards (0 to 9)
- symbol cards (=, <, >, ≤, ≥, +, −, ×, ÷, ≠)
- shape cards (◯, △, □, ⬠, ⬡ / large and small / black and white
- number lines (−9 to 0, 0 to 9, 10 to 20, 0 to 100)
- number strips or rods or plastic cubes stuck together
- dominoes
- toy money
- coloured counters
- coloured sticks/straws

Reference is made to a number of A2 posters *. We would also like you to encourage learners to collect their own materials for use in maths lessons (e.g. pebbles, shells, buttons, sweets, trinkets) to make maths more relevant to them individually.

* online at http://www.cimt.org.uk/projects/mepres/primary/index.htm

Year	Week beginning	*Order of Topics*
	Sept	Familiarisation with Book 3 resources.
		1. Revision: numbers up to 100; addition and subtraction
		2. Revision: Multiplication
		3. Revision: Division
	Oct	4. Revision: Division with remainders; 4 operations
		5. Revision: length, capacity, mass, time.
		6. Collecting, recording and interpreting data; lists, tables, graphs
		Half Term
	Nov	7. Numbers up to 200; addition and subtraction
		8. Numbers up to 200: multiplication and division
		9. Numbers up to 200: 4 operations
		10. Order of calculation; rounding to the nearest 10
		11. 2-D and 3–D shapes, parallel and perpendicular lines. Fractions: half, quarter, third
	Dec	12: Time: quarter, half, three quarters of an hour. Introduction to the 24 hour clock
		13. Fractions: contextual problems. Revision and practice.
		Christmas
	Jan	14. Extending numbers to 1000
		15. Operations with whole tens and hundreds up to 1000. Roman numerals. Sequences
		16. Rounding to tens and hundreds. Measuring length. Changing units.
		17. Measurement: calculation with quantities. Capacity and money
	Feb	18. Estimation. Vertical addition without crossing tens (H, T, U)
		19. Problems in context. Column addition, crossing tens (Th, H, T, U)
		Half Term
		20. Estimating differences. Column subtraction.
	Mar	21. Addition and subtraction. Problems in context
		22. Geometry: sorting 1-D, 2-D, 3-D shapes. Compass directions. Right angle turns.
		23. Reflection, symmetry. Parallel and perpendicular lines.
		24. Enlargements, reductions. Building 3-D shapes
		25. Fractions: using and finding halves, quarters, eighths and thirds
	Apr	**Easter**
		26. Revision and practice. Equations, inequalities. Problems in context. Negative numbers.
		27. Multiplication. Estimation of products.
	May	28. Multiplication without tens crossing. Problems in context.
		29. Graphs. Perimeter and area of rectangles. Quantities: mass, capacity, length, time
		30. Division. Divisor, factor, multiple
		31. Division: problems in context. Probability: simple experiments
		Half Term
	Jun	32. Roman numerals. Money problems.
		33. Revision: enlargement, reduction, similarity, perimeter, area
		34. Building and drawing solids. Probability: combinatorics. Numbers up to 10 000.
		35. Revision and practice: length, capacity, mass. Decimal number system. Puzzles, challenges
	Jul	

Book 3 Contents Page
Lessons 1 to 30

R: Revision- Not included in the contents page **C: Core** *E: Extension*

Book 3 Contents Page
Lessons 31 to 50

R: Revision- Not included **C: Core** *E: Extension*

Book 3 Contents Page
Lessons 51 to 80

R: Revision- Not included **C: Core** *E: Extension*

Year 3 Contents Page
Lessons 81 to 110

R: Revision- Not included **C: Core** *E: Extension*

Book 3 Contents Page
Lessons 111 to 140

R: Revision- Not included **C: Core** *E: Extension*

Book 3 Contents Page
Lessons 141 to 175

R: Revision- Not included **C: Core** *E: Extension*